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Optimization of the Linear Pulsed Heat Source Method and Basic Structural Dimensions of the Device for Measuring Thermophysical Properties of Solid Materials to Improve the Laboratory Management System

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Abstract

Using the methods of metrology and the theory of thermal conductivity, the mathematical models of relative errors in measuring the volumetric heat capacity and thermal diffusivity of solid materials were developed using the linear pulsed heat source method, which created a method for choosing the optimal conditions for processing the experimental data, the main structural dimensions of the measuring device, and optimal duration of the heat pulse.

Keywords

Thermal diffusivity; volumetric heat capacity; measurement; relative errors; minimization; heat pulse; data processing; structural dimensions; optimization.

Introduction

In the conditions of rapid development of technologies and creation of new materials, it is important to study their thermophysical properties (**TPPs**). In the last decade, research into the development and modernization of new methods for implementing the so-called "instantaneous" sources of heat (moisture) has been quite actively conducted [1–16].

Traditional methods of implementing methods of "instantaneous" heat sources did not pay enough attention to the choice of (1) optimal conditions for measuring and processing primary information; (2) the rational structural dimensions of the measuring devices used; (3) the actual duration of the heat pulse τ_p [1–4]. Only recently, publications [5–10] addressed the issues of optimizing the operating parameters of the measurement process and the rational values of the structural dimensions of measuring devices, but the question of choosing the optimal duration of a thermal pulse was not discussed.

The purpose of this article is to come up with recommendations on choosing the best (optimal from the point of view of minimizing the errors in measuring TPPs): (1) conditions for processing the data obtained during the experiment; (2) distance between the heater and the temperature meter; (3) duration of the heat pulse supplied to the linear heater.

To achieve this goal the following problems were set and solved:

(1) mathematical formulation of the problem of choosing the optimal conditions for the experiment and subsequent processing of the experimental data for the linear pulsed heat source method were proposed;

(2) the problem of choosing (a) the optimal duration of the heat pulse; (b) the main structural dimensions of the measuring device; (c) parameters of the experimental data processing algorithm was solved;

(3) recommendations on the implementation of the linear pulsed heat source method when measuring TPPs of solid materials were given.

In the mathematical model of the temperature field $T(r, \tau)$ in radial coordinate system:

$$c\rho\frac{\partial T(r,\tau)}{\partial\tau} = \lambda\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T(r,\tau)}{\partial r}\right] + W(r,\tau);$$

$$\tau > 0;\ 0 < r < \infty; \tag{1}$$

$$T(r,0) = T_0 = 0; \tag{2}$$

$$\frac{\partial T(0,\tau)}{\partial r}=0; \quad (3)$$

$$T(\infty,\tau)=T_0=0 \quad (4)$$

in which the internal heat source $W(r, \tau)$ was previously set as a linear instantaneous pulse

$$W(r,\tau)=Q_{\text{lin}}\,\delta(r)\delta(\tau),$$

to achieve the goal formulated above, a source of heat in the form of a heat pulse with a duration of τ_p is set

$$W(r,\tau)=q_{\text{lin}}\,\delta(r-0)\left[h(\tau-0)-h\left(\tau-\tau_\text{p}\right)\right], \quad (1a)$$

providing for the supply of heat to the linear heater for a period of time $0\le\tau\le\tau_\text{p}$.

The above designations are as follows: r, τ is the spatial coordinate of the sample and time; $c\rho$, λ are volumetric heat capacity and thermal conductivity of the material under study; T_0 is initial material temperature (at time $\tau = 0$), assuming that the start of temperature scale in each experiment i.e. $T_0 = 0$; Q_lin is the total amount of heat released in a unit length of a linear heater if $r = 0$ at time $\tau = 0$; $\delta(r)$, $\delta(\tau)$ are Dirac's symbolic delta functions [3, 11, 12], is τ_p the duration of the real (not instantaneous) heat pulse supplied to the heater; $q_\text{lin}=Q_\text{lin}/\tau_\text{p}$ is the amount of heat released by the unit of length of the linear source of heat per unit of time; $h(\tau-0)$, $h\left(\tau-\tau_\text{p}\right)$are single step functions [11, 12].

The physical model of the measuring device

The physical model of the measuring device is a cell (Fig. 1) into which a sample consisting of two plates is placed – the bottom one and the top one. A linear electric heater (e.g. made in the form of thin metal wire made of nichrome, manganin or constantan) with a length L is placed between the upper face of the lower plate and the lower face of the upper plate, while a temperature meter (in the form of a resistance thermometer made from copper or tungsten wire or in the form of a thermocouple) is placed at the distance r from the heater in the same plane. Diagrams and designs of similar measuring devices were considered in [1–6, 13, 19, 20, 22].

Presented in Fig. 1 the measuring cell includes the following main elements:

– a sample of the material made in the form of two plates *1*, *2*. Heights L_1 and L_2 along the axis x of the plates *1* and *2* the heat-insulating material under investigation have to be about 60 mm;

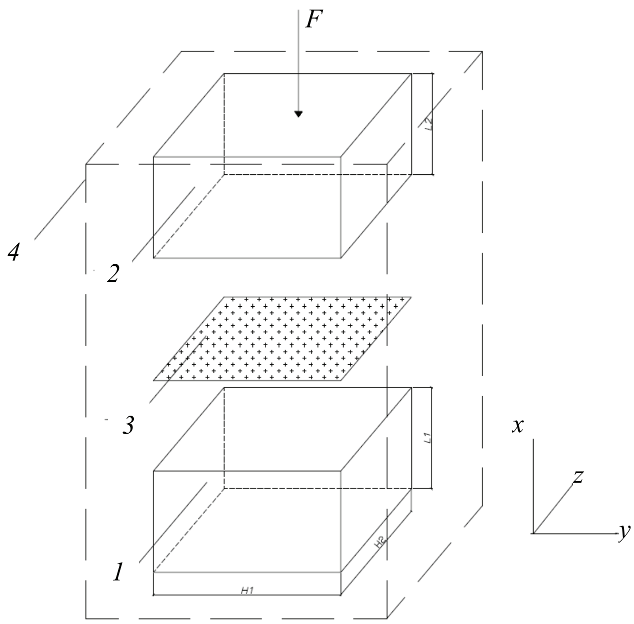

Fig. 1. Measuring cell of the pulsed linear heat source method with the mutual arrangement of its components

– the measuring plate *3*, consisting of a frame on which two wire elements are stretched (heater and temperature meter), the design of which will be described below;

– easily removable thermal insulation conditionally shown in Fig. 1 as dashed lines *4*. This easily removable insulation is made of foam plastic in the form of three component parts, the internal dimensions of which are 2–3 mm higher than the external dimensions (along the y, z axes) of plates *1* and *2* of the sample of the material under study;

– to reduce the thermal resistances that occur at the points of contact of the plates *1* and *2* of the sample between themselves and with the wire elements of the measuring plate *3*, the design of the measuring cell involves the use of a constant-weight load, which creates the force F shown in Fig. 1 by an arrow, and ensuring mutual pressing of plates *1* and *2* to each other with a constant force, which allows stabilizing the value of thermal resistances and minimizing the effect of their changes on the measurement results of the desired thermophysical properties, namely, thermal diffusivity a, volumetric heat capacity $c\rho$ and thermal conductivity $\lambda = a\, c\rho$.

The design of the measuring plate (a holder of the linear heater and temperature meter)

The method used is based on the use of two thin wires *1* and *2* (a linear heater and a temperature meter), stretched over a strong, but rather thin frame *3* (Fig. 2).

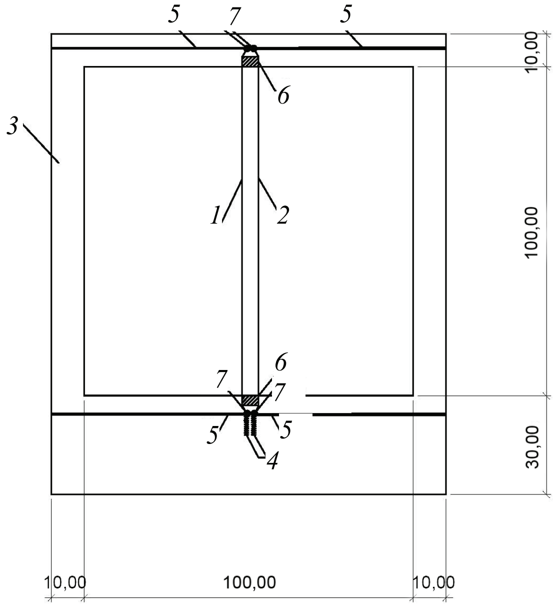

Fig. 2. Drawing of the measuring plate for the linear pulsed heat source method

The frame *3* of the measuring plate of the linear pulsed heat source method consists of a dielectric material (textolite) measuring 120 by 140 mm and a thickness of 2–3 mm. Inside it is cut a square hole 100 by 100 mm in size, in the middle of which two wire elements *1* and *2* are stretched. The linear heater *1*, madein the form of thin metal wire made of nichrome, manganin or constantan, heats the sample surface when a pulse τ_p, is applied to it, and a temperature meter *2* is placed at a distance *r* from the heater (in the form of a resistance thermometer (made of copper or tungsten wire) or thermocouple). The distance *r* between the wire elements *1* and *2* is set (and adjusted) by the braces *6*. The heater *1* and the temperature meter *2* are fixed on the frame *3* by means of springs *4*. The wires *1* and *2* are soldered at both ends to insulated copper wires *5* of large cross section. Solder joints are marked with *7*.

The mathematical model of the temperature field

The mathematical model of the temperature field $T(r, \tau)$ in the material under study (in the case of a pulsed linear heat source) can be written in the form (1) – (4) with a modified function of the source of heat $W(r, \tau)$, given by the formula (1a).

The solution of the boundary value problem (1) – (4) with a continuously operating constant source of heat $W(r,\tau)=q_{\text{lin}}\delta(r-0)h(\tau-0)$, has the form [2]

$$[T(r,\tau)-T_0]=-\frac{q_{\text{lin}}}{4\pi\lambda}Ei\left(-\frac{r^2}{4a\tau}\right), \qquad (5)$$

where $q_{\text{lin}}=Q_{\text{lin}}/\tau_p$ is the amount of heat released by the unit of length of the linear source of heat per unit of time; $a=\lambda/c\rho$ is temperature conductivity coefficient; $h(\tau-0)$ is single step function [16, 17], having the form:

$$h(\tau-0)=\begin{cases}0 \text{ over } \tau<0;\\ 1 \text{ over } \tau\ge 0.\end{cases}$$

If (in the case of a pulsed heat source considered in the article, the duration $0<\tau\le\tau_p$) wet $W(r,\tau)=q_{\text{lin}}\delta(r-0)[h(t)-h(t-\tau_p)]$, in the basis of the principle of superposition and the well-known solution (5), we obtain that if $\tau\ge\tau_p$:

$$[T(r,\tau)-T_0]=-\frac{q_{\text{lin}}}{4\pi\lambda}\left[Ei\left(-\frac{r^2}{4a\tau}\right)-Ei\left(-\frac{r^2}{4a(\tau-\tau_p)}\right)\right]. \qquad (6)$$

Thus, the general solution of problem (1) – (4) taking into account (5) and (6) takes the form:

$$[T(r,\tau)-T_0]=\begin{cases}-\dfrac{q_{\text{lin}}}{4\pi\lambda}Ei\,(-U(\tau)) \text{ over } 0<\tau<\tau_p;\\ -\dfrac{q_{\text{lin}}}{4\pi\lambda}\left[Ei\,(-U(\tau))-Ei(-U(\tau-\tau_p))\right]\\ \qquad\qquad\qquad \text{over } \tau\ge\tau_p,\end{cases} \qquad (7)$$

where $Ei(u)=\int_{-\infty}^{u}\frac{e^t}{t}dt=-\int_{-u}^{\infty}\frac{e^{-t}}{t}dt$ is integral exponential function [2, 11, 12]; $U(\tau)=\frac{r^2}{4a\tau}$, $U(\tau-\tau_p)=\frac{r^2}{4a(\tau-\tau_p)}$ are dimensionless functions depending on r, τ, τ_p, a, and

$$U(\tau-\tau_p)=\frac{r^2}{4a(\tau-\tau_p)}=\frac{r^2}{4a\tau\left(\dfrac{\tau-\tau_p}{\tau}\right)}=U(\tau)\frac{\tau}{\tau-\tau_p}.$$

The traditional approach to the experiment and the subsequent processing of the data

The traditional approach to the experiment and the subsequent processing of the data when measuring TPPs using the linear "instantaneous" heat source consists of several steps [1–4]:

(1) manufacture of two massive plates from the material under study (their thickness should be less than ten to twenty times the distance r between the electric heater and the temperature meter);

(2) placing the heater and temperature meter at a distance r from each other between the two plates, and then achieving a uniform distribution of the temperature field $T(r, \tau) = T_0 = \text{const}$ inside the sample of the material under study;

(3) supply of constant power P for a specified short period of time $0 < \tau \le \tau_p$ to a linear electric heater of length L and recording the change in temperature difference over time $[T(r,\tau)-T_0]$ by a signal from a temperature meter;

(4) determination from the obtained experimental data of the maximum value of the temperature difference $[T_{max}-T_0]=[T(r,\tau_{max})-T_0]$ and the value of the moment of time $\tau = \tau_{max}$, corresponding to this maximum value $[T_{max}-T_0]$, as well as the amount of heat $Q_{lin}=q_{lin}\tau_p$, released in a unit of length of the heater;

(5) calculation by the obtained values τ_{max}, $[T_{max}-T_0]$, taking into account the known r, Q_{lin}, the desired values of thermal diffusivity a and thermal conductivity λ of the material under study.

For the traditional order of the experiment it is typical to have:

1) high relative error in determining the point in time $\tau = \tau_{max}$ (about 15–20 %),

2) lack of recommendations on selecting:

– optimal conditions for processing experimental data;

– optimal distance r between the linear heater and the temperature meter;

– the optimal value of the duration τ_p of the thermal pulse.

The method of processing experimental data

The method of processing experimental data proposed in this article is based on the use of a dimensionless parameter

$$\gamma = \frac{T(r,\tau)-T_0}{T_{max}-T_0}, \tag{8}$$

which is a temperature difference relation $[T(r,\tau)-T_0]$ (at time τ) to maximum temperature difference $[T(r,\tau_{max})-T_0]=[T_{max}-T_0]$, taking place at time $\tau = \tau_{max}$.

Moreover, each value of the temperature difference $\gamma\,[T_{max}-T_0]=[T(r,\tau)-T_0]$, that is, each value of the dimensionless parameter γ corresponds to a specific value of time τ.

In the mathematical modeling of measuring TPPs first with a constant step $\Delta\tau$ in time τ by formula (7), the values of temperature differences $[T(r,\tau_i)-T_0]$ and relevant times τ_i, $i = 1, 2, \ldots, n$, were calculated and recorded (in the form of arrays), and then by the data array $[T(r,\tau_i)-T_0]$, $i = 1, 2, \ldots, n$, the maximum value $[T_{max}-T_0]$ of this difference was found

$$[T(r,\tau_{max})-T_0]= = \frac{q_{lin}}{4\pi\lambda}\left\{Ei\left[-U(\tau_{max})\frac{\tau_{max}}{\tau_{max}-\tau_p}\right]-Ei[-U(\tau_{max})]\right\}. \tag{9}$$

After this, the method of interpolation, the value of time τ, corresponding to the value of the temperature difference $[T(r,\tau)-T_0]=\gamma\,[T_{max}-T_0]$ was found.

Dividing the dependence (7) for $\tau > \tau_p$ by (9), we obtain that

$$\gamma = \frac{T(r,\tau)-T_0}{T_{max}-T_0} = \frac{Ei\left[-U(\tau)\frac{\tau}{\tau-\tau_p}\right]-Ei[-U(\tau)]}{Ei\left[-U(\tau_{max})\frac{\tau_{max}}{\tau_{max}-\tau_p}\right]-Ei[-U(\tau_{max})]}. \tag{10}$$

If the duration of τ_p of heat pulse is known from the experiment, temperature difference values $[T(r,\tau_i)-T_0]$, and the corresponding values of the moments of time τ_i, $i = 1, 2, \ldots, n$, then by solving equation (10) we find the value of the dimensionless value

$$U(\tau') = \frac{r^2}{4a\tau'}, \tag{11}$$

corresponding to the specified value of the parameter γ, moreover, the value of the moment of time $\tau' = \tau'(\gamma)$ is a function of the value of the parameter γ.

It follows from (11) that the calculated ratio for calculating the coefficient of thermal diffusivity is

$$a = \frac{r^2}{4\tau' U(\tau')}. \tag{12}$$

After transforming relation (6), a formula was obtained for calculating the volumetric heat capacity

$$c\rho = \frac{q_{\text{lin}}\tau}{\pi r^2 [T(r,\tau) - T_0]} \times$$

$$\times U(\tau)\left\{ Ei\left[-U(\tau)\frac{\tau}{\tau - \tau_{\text{p}}} \right] - Ei[-U(\tau)] \right\}. \quad (13)$$

Having obtained the formulas (12) and (13), the question arises: “At what value of the dimensionless parameter γ will the minimum errors of measurement of the desired a and $c\rho$ of thermal diffusivity coefficient and volumetric heat capacity take place?”.

The mathematical model of root-mean-square (RMS) estimation of relative errors $(\delta a)_{\text{RMS}}$ in the measurement of the thermal diffusivity

According to the theory of errors [3, 14, 15], after the logarithm of dependence (12) and the subsequent determination of the differential from the left and right parts (by analogy with the one stated in [3, 5, 6, 9, 10, 13–22]), we obtain:

$$\ln a = 2\ln r - \ln 4 - \ln \tau - \ln U(\tau);$$

$$d\ln a = 2d\ln r - d\ln 4 - d\ln \tau - d\ln U(\tau);$$

$$\frac{da}{a} = 2\frac{dr}{r} - \frac{d4}{4} - \frac{d\tau}{\tau} - \frac{dU(\tau)}{U(\tau)}.$$

According to the theory of errors [3, 14, 15] we substitute differentials $da \approx \Delta a$, $dr \approx \Delta r$, $d\tau \approx \Delta\tau$, $dU(\tau) \approx \Delta U(\tau)$ with absolute errors Δa, Δx, $\Delta\tau$, $\Delta U(\tau)$ and obtain

$$\frac{\Delta a}{a} = 2\frac{\Delta r}{r} - \frac{\Delta\tau}{\tau} - \frac{\Delta U(\tau)}{U(\tau)},$$

where it is taken into account that the differential constant $d4 = 0$.

Substituting the signs “–”with the signs “+”, we obtain [3, 14, 15] the expression for calculating the so-called marginal estimate of the relative error in measuring the coefficient of thermal diffusivity $\left(\frac{\Delta a}{a}\right)_{\text{marg}} = 2\frac{\Delta r}{r} - \frac{\Delta\tau}{\tau} - \frac{\Delta U(\tau)}{U(\tau)}$ or $(\delta a)_{\text{marg}} = 2\delta r + \delta\tau + + \delta U(\tau)$, where $\delta a_{\text{marg}} = \frac{\Delta a}{a}$, $\delta r = \frac{\Delta r}{r}$, $\delta\tau = \frac{\Delta\tau}{\tau}$, $\delta U(\tau) = \frac{\Delta U(\tau)}{U(\tau)}$ – relative errors in determining the corresponding physical values a, r, τ, $U(\tau)$.

After the transition (by analogy with the recommendations [3, 5, 6, 9, 10, 13–22]) from the limit $(\delta a)_{\text{marg}}$ to the mean-square estimate $(\delta a)_{\text{RMS}}$ of the relative error in determining the thermal diffusivity, we obtain

$$(\delta a)_{\text{RMS}} = \sqrt{4(\delta r)^2 + (\delta\tau)^2 + [\delta U(\tau)]^2}. \quad (14)$$

Let us consider in more detail the procedure for determining the errors included in the last expression (14). Taking into account that the value of the moment of time depends on the dimensionless parameter γ, that is $\tau = \tau(\gamma)$, we obtain

$$\delta[U(\tau(\gamma))] = \delta U(\gamma) \approx \frac{dU(\gamma)}{U(\gamma)} = \frac{1}{U}\frac{dU}{d\gamma}d\gamma \approx \frac{1}{U}\frac{dU}{d\gamma}\Delta\gamma.$$

To determine the absolute error $\Delta\gamma$ we perform (by analogy with the above) the transformations of formula (8) and obtain

$$(\delta\gamma)_{\text{RMS}} = \sqrt{\left[\frac{\Delta T}{T(r,\tau) - T_0}\right]^2 + \left[\frac{\Delta T}{T_{\max} - T_0}\right]^2} =$$

$$= \delta(T_{\max} - T_0)\sqrt{\frac{1}{\gamma^2} + 1}$$

or

$$\Delta\gamma = \gamma\delta(T_{\max} - T_0)\sqrt{\frac{1}{\gamma^2} + 1} = \delta(T_{\max} - T_0)\sqrt{1 + \gamma^2}, \quad (15)$$

where ΔT is absolute measurement error of temperature differences; $\delta(T_{\max} - T_0)$ is relative error of measurement of the maximum value of the temperature difference $(T_{\max} - T_0)$; $\Delta\gamma$, $(\delta\gamma)_{\text{RMS}}$ are absolute and root mean square relative error in determining the dimensionless parameter γ from the experimentally measured values of temperature differences $[T(r,\tau) - T_0]$ and $[T_{\max} - T_0]$.

Included in (14) the relative error $\delta\tau$ determining the value of the point in time τ is related to the errors in measuring the temperature differences $[T(r,\tau) - T_0]$. From the expression $\frac{\partial[T(r,\tau) - T_0]}{\partial\tau} \approx \frac{\Delta T}{\Delta\tau}$, we obtain

$$\delta\tau = \frac{\Delta\tau}{\tau} = \frac{\Delta T}{\tau\left\{\dfrac{\partial[T(r,\tau)-T_0]}{\partial\tau}\right\}}, \qquad (16)$$

where Δτ, δτ are absolute and relative errors of determining the value of the point of time τ, corresponding to the given value of the dimensionless parameter γ.

Substituting (15), (16) into formula (14), we obtain the relation

$$(\delta a)_{\mathrm{RMS}} = \sqrt{4(\delta r)^2 + \left[\frac{\Delta T}{\tau\left\{\dfrac{\partial[T(r,\tau)-T_0]}{\partial\tau}\right\}}\right]^2 + \left[\frac{1}{U(\tau(\gamma))}\frac{dU(\tau(\gamma))}{d\gamma}\sqrt{\gamma^2+1}\,\delta(T_{\max}-T_0)\right]^2}, \qquad (17)$$

used by us in further calculations in order to identify the optimal value of the dimensionless parameter γ (when measuring thermal diffusivity a).

The mathematical model of RMS estimation of relative errors $(\delta c\rho)_{\mathrm{RMS}}$ in measuring the volumetric heat capacity

Formula (13) can be represented as

$$c\rho = \frac{q_{\mathrm{lin}}\tau}{\pi r^2[T(r,\tau)-T_0]}F[U(\tau)], \qquad (18)$$

where $F[U(\tau)] = U(\tau)\left\{Ei\left[-U(\tau)\frac{\tau}{\tau-\tau_{\mathrm{p}}}\right] - Ei[-U(\tau)]\right\}$.

Then, according to the theory of errors [3, 14, 15], after taking the logarithm function (18) and subsequent determination of the differentials of the left and right sides, we obtain:

$$\ln c\rho = \ln q_{\mathrm{lin}} + \ln\tau + \ln F[U(\tau)] - \\ -\ln\pi - 2\ln r - \ln[T(r,\tau)-T_0];$$

$$d\ln c\rho = d\ln q_{\mathrm{lin}} + d\ln\tau + d\ln F[U(\tau)] - \\ -d\ln\pi - 2d\ln r - d\ln[T(r,\tau)-T_0];$$

$$\frac{d\,c\rho}{c\rho} = \frac{dq_{\mathrm{lin}}}{q_{\mathrm{lin}}} + \frac{d\tau}{\tau} + \frac{dF[U(\tau)]}{F[U(\tau)]} - \frac{d\pi}{\pi} - 2\frac{dr}{r} - \frac{d[T(r,\tau)-T_0]}{[T(r,\tau)-T_0]}.$$

According to the theory of errors [3, 14, 15] we substitute differentials $dc\rho \approx \Delta c\rho$, $dr \approx \Delta r$, $d\tau \approx \Delta\tau$, $dq_{\mathrm{lin}} \approx \Delta q_{\mathrm{lin}}$, $dF[U(\tau)] \approx \Delta F[U(\tau)]$, with absolute errors $\Delta c\rho$, Δr, $\Delta\tau$, Δq_{lin}, $\Delta U(\tau)$, $\Delta F[U(\tau)]$, and obtain

$$\frac{\Delta c\rho}{c\rho} = \frac{\Delta q_{\mathrm{lin}}}{q_{\mathrm{lin}}} + \frac{\Delta\tau}{\tau} + \frac{\Delta F[U(\tau)]}{F[U(\tau)]} - 2\frac{\Delta r}{r} - \frac{\Delta[T(r,\tau)-T_0]}{[T(r,\tau)-T_0]},$$

where it is taken into account that the differential constant $d\pi = 0$ having substituted the signs "–" with the signs "+", we obtain an expression for calculating the so-called marginal estimate of the relative error [3, 14, 15] of measuring the volumetric heat capacity

$$\left(\frac{\Delta c\rho}{c\rho}\right)_{\mathrm{marg}} = \frac{\Delta q_{\mathrm{lin}}}{q_{\mathrm{lin}}} + \frac{\Delta\tau}{\tau} + \frac{\Delta F[U(\tau)]}{F[U(\tau)]} + 2\frac{\Delta r}{r} + \frac{\Delta[T(r,\tau)-T_0]}{[T(r,\tau)-T_0]}$$

or

$$(\delta c\rho)_{\mathrm{marg}} = \delta q_{\mathrm{lin}} + \delta\tau + \delta F[U(\tau)] + 2\delta r + \delta[T(r,\tau)-T_0], \qquad (19)$$

where $\delta c\rho_{\mathrm{marg}} \approx \frac{\Delta c\rho}{c\rho}$, $\delta r \approx \frac{\Delta r}{r}$, $\delta\tau \approx \frac{\Delta\tau}{\tau}$, $\delta q_{\mathrm{lin}} \approx \frac{\Delta q_{\mathrm{lin}}}{q_{\mathrm{lin}}}$, $\delta F[U(\tau)]$ are relative errors in determining the corresponding physical values $c\rho$, r, τ, $U(\tau)$, q_{lin}.

Acting by analogy with the above procedure for determining errors, we obtain:

$$\delta F[U(\tau(\gamma))] = \frac{1}{F[U(\tau(\gamma))]}\frac{\partial F[U(\tau(\gamma))]}{\partial\gamma}\Delta\gamma;$$

$$\Delta\gamma = \gamma\,\delta(T_{\max}-T_0)\sqrt{\frac{1}{\gamma^2}+1} = \delta(T_{\max}-T_0)\sqrt{1+\gamma^2};$$

$$\delta[T(r,\tau)-T_0] = \frac{\Delta T}{T(r,\tau)-T_0};$$

$$\delta\tau = \frac{\Delta T}{\tau\left(\dfrac{\partial[T(r,\tau)-T_0]}{\partial\tau}\right)}.$$

With this in mind, the expression (19) takes the form

$$(\delta c\rho)_{\text{marg}} = \delta q_{\text{lin}} + \frac{\Delta T}{\tau\left(\frac{\partial[T(r,\tau)-T_0]}{\partial\tau}\right)} + \frac{1}{F[U(\tau(\gamma))]}\frac{\partial F[U(\tau(\gamma))]}{\partial\gamma}\delta(T_{\max}-T_0)\sqrt{1+\gamma^2} + 2\delta r + \frac{\Delta T}{T(r,\tau)-T_0}.$$

After the transition (by analogy with the recommendations [3, 6, 7, 10, 14–20, 22]) from the limit estimate $(\delta c\rho)_{\text{marg}}$ to the mean-square estimate $(\delta c\rho)_{\text{RMS}}$ of the error in measuring the volumetric heat capacity, we obtain

$$(\delta c\rho)_{\text{RMS}} = \\ = \sqrt{(\delta q_{\text{lin}})^2 + \left\{\frac{\Delta T}{\tau\left(\frac{\partial[T(r,\tau)-T_0]}{\partial\tau}\right)}\right\}^2 + \left\{\frac{1}{F[U(\tau(\gamma))]}\frac{\partial F[U(\tau(\gamma))]}{\partial\gamma}\delta(T_{\max}-T_0)\sqrt{1+\gamma^2}\right\}^2 + 4(\delta r)^2 + \left\{\frac{\Delta T}{T(r,\tau)-T_0}\right\}^2}. \quad (20)$$

Finding a formula for choosing the optimal value of the duration τ_p of heat pulse supplied to the linear heater

In the process of the research, it became apparent that the relative mean-square errors $(\delta a)_{\text{RMS}}$ of measuring the thermal diffusivity *a* and $(\delta c\rho)_{\text{RMS}}$ of measuring the volumetric heat capacity $c\rho$ additionally depend on the duration τ_p and the thermal impulse.

When making measurements, it is desirable to ensure that the requirement for supplying a linear heater of such a value of power *P*, at which the maximum temperature difference $[T(r, \tau_{\max}) - T_0] = [T_{\max} - T_0]$ achieved at the moment of time $\tau = \tau_{\max}$ during each experiment distance *r* from the heater remains approximately the same and is within certain limits, which is necessary for the following reasons:

– if this maximum difference $[T_{\max} - T_0]$ is small, then the relative error of measuring the values of temperature differences $[T(r, \tau) - T_0]$ will be too large, which can lead to an increase in the relative errors $(\delta a)_{\text{RMS}}$, $(\delta c\rho)_{\text{RMS}}$ in measurement of the desired thermophysical properties (**TPPs**);

– if this maximum $[T_{\max} - T_0]$ is too big, the assumption that the heat transfer processes in the sample described by a linear mathematical model (1) – (4) is not satisfied, which in turn will lead to increased resulting errors $(\delta a)_{\text{RMS}}$, $(\delta c\rho)_{\text{RMS}}$ in measurement of the desired thermophysical properties due to nonlinearities that are not taken into account by the linear boundary value problem (1) – (4).

To fulfill the requirement (that $[T_{\max} - T_0] \approx \approx \text{const}$), at each value of the duration τ_p of heat pulse linear heater must ensure the creation of power density $q_{\text{lin}} = \frac{P}{L}$, at which a constant total amount of heat is emitted within the sample per unit length of the heater in each experiment

$$Q_{\text{lin}} = q_{\text{lin}}\tau_p = \text{const}, \quad (21)$$

where q_{lin} is power density supplied by the heater power *P* and length *L* to the sample during the time period $0 \le \tau \le \tau_p$.

The numerical calculations showed that in the study of samples of solid materials with a distance from the heater to the temperature meter $3 \le r \le 6$ mm to obtain the temperature difference $[T_{\max} - T_0] = 3$–7 °C, total amount of heat Q_{lin}, released in a unit of length *L* of the electric heater should be maintained within $Q_{\text{lin}} = \frac{P}{L}\tau_p = 1000$–$2000$ J/m. Therefore, as shown in Fig. 3*a*, *b*, the data and results of determining the optimal value τ_p^{opt}, ensuring minimum errors $(\delta a)_{\text{RMS}}$ and $(\delta c\rho)_{\text{RMS}}$ of measurement *a* and $c\rho$, were obtained for $Q_{\text{lin}} = 1500$ J/m.

Let us consider the calculation of the error δq_{lin}, included in the formula (20) in more detail. From the above it follows

$$q_{\text{lin}} = \frac{Q_{\text{lin}}}{\tau_p} \text{ and } q_{\text{lin}} = \frac{P}{L}, \quad (22)$$

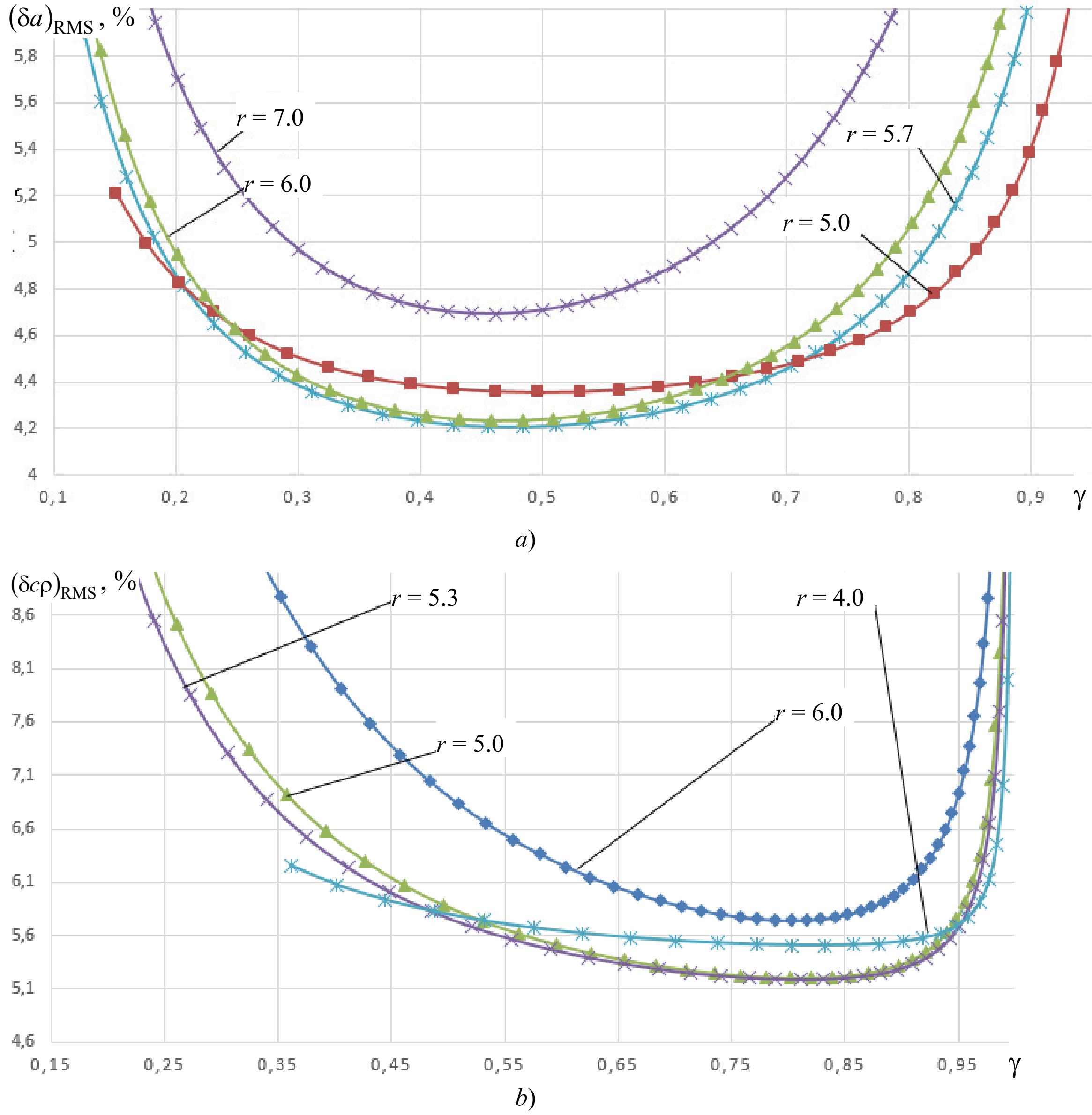

Fig. 3. Dependencies of root-mean-square relative errors $(\delta a)_{\text{RMS}}$ and $(\delta c\rho)_{\text{RMS}}$ on the dimensionless parameter γ for different values of the distance *r* from the linear pulsed source of heat to the temperature meter during the measurement: *a* – coefficient of thermal diffusivity *a*; *b* – volumetric heat capacity *c*ρ

i.e. $\frac{P}{L}=\frac{Q_{\text{lin}}}{\tau_{\text{p}}}$. At the same time electrical power *P*, supplied to the flat heater should be chosen from the ratio $P=\frac{Q_{\text{lin}}L}{\tau_{\text{p}}}$, and, for $Q_{\text{lin}}=1500$ J/m, $L=0.1$ m it turns out that

$$P(\tau_{\text{p}})=\frac{150}{\tau_{\text{p}}}. \qquad (23)$$

After logarithm (22), the definition of the differentials from the left and right parts of the resulting ratio and the implementation of other recommendations of the theory of errors [3, 6, 7, 10, 14–20, 22], we obtain the formula

$$\delta q_{\text{lin}}=\sqrt{(\delta P)^2+(\delta L)^2}=$$

$$=\sqrt{\left[\frac{\Delta P}{P(\tau_{\text{p}})}\right]^2+\left[\frac{\Delta L}{L}\right]^2}, \qquad (24)$$

in which the value $P(\tau_{\text{p}})$ was calculated by the formula (23).

After substituting (24) into (20), we obtain the formula

$$(\delta c\rho)_{\mathrm{RMS}} =$$

$$= \sqrt{\left[\frac{\Delta P}{P(\tau_{\mathrm{p}})}\right]^2 + \left[\frac{\Delta L}{L}\right]^2 + \left\{\frac{1}{F[U(\tau(\gamma))]}\frac{\partial F[U(\tau(\gamma))]}{\partial \gamma}\delta(T_{\max} - T_0)\sqrt{1+\gamma^2}\right\}^2 + 4(\delta r)^2 + \left\{\frac{\Delta T}{\tau\left(\frac{\partial[T(r,\tau)-T_0]}{\partial \tau}\right)}\right\} + \left\{\frac{\Delta T}{T(r,\tau)-T_0}\right\}^2} \; . \tag{25}$$

The results of numerical modeling of the root-mean-square estimates of the relative errors in the measurement of the thermal diffusivity *a* and the volumetric heat capacity *cρ*

Using the obtained formulas (17) and (25), we calculated the dependences on the dimensionless parameter γ of the root-mean-square relative errors $(\delta a)_{\mathrm{RMS}}$, $(\delta c\rho)_{\mathrm{RMS}}$, for the duration of the heat pulse $\tau_{\mathrm{p}} = 21$ s. The calculation results are presented in Fig. 3. At the same time, the following initial data were used in the calculations: $a = 1.06 \cdot 10^{-7}$ m^2/s, $c\rho = 1.83 \cdot 10^6$ J/(m^3·K), $\Delta P = 0.1$ W, $r = 2–8$ mm, $\Delta r = 0,1$ mm, $\Delta T = 0.05$ K, $\Delta L/L = \delta L = 0.5$ %.

Fig. 3 shows that the minimum values of relative errors $(\delta a)_{\mathrm{RMS}}$, $(\delta c\rho)_{\mathrm{RMS}}$ depend not only on the parameter γ, but also on the value of the distance r between the linear pulse heater and the temperature meter. In this regard, it was decided to build lines of equal error levels on the plane with coordinates (γ, r) for the duration of the heat pulse $\tau_{\mathrm{p}} = 21$ s. The results of this work are presented in Fig. 4.

The results of calculations presented in Fig. 4 show that (with the initial data used in the calculations) the minimum values of the root-mean-square relative errors $(\delta a)_{\mathrm{RMS}}$ of measuring the thermal diffusivity a are achieved with the values of the dimensionless parameter γ^a in the range $0.41 < \gamma^a \le 0.55$ and with the values of the main structural dimension of the measuring device within $5.4 < r < 6.0$ mm, and $\gamma^a_{\mathrm{opt}} \approx 0.48$, $r^a_{\mathrm{opt}} \approx 5.7$ mm.

At the same time, the minimum values of the mean square relative errors $(\delta c\rho)_{\mathrm{RMS}}$ of measuring the volumetric heat capacity $c\rho$ occur at $0.78 < \gamma^{c\rho} \le 0.84$ and $4.5 < r < 5.1$ mm, and $\gamma^{c\rho}_{\mathrm{opt}} \approx 0.81$, $r^{c\rho}_{\mathrm{opt}} \approx 4.8$ mm.

Thus, to achieve the minimum values of error $(\delta a)_{\mathrm{RMS}}$ and $(\delta c\rho)_{\mathrm{RMS}}$ in measuring the thermal diffusivity a and volumetric heat capacity $c\rho$ of the material under study, a measuring transducer should be used with the distance between the temperature meter and the heater in the range $5.1 < r < 5.4$ mm, and, you can take

$$r_{\mathrm{opt}} = \frac{r^a_{\mathrm{opt}} + r^{c\rho}_{\mathrm{opt}}}{2} = 5.25 \text{ mm.}$$

To determine the optimal value of the duration τ_{p} of heat pulse, ensuring the achievement of the minimum values of relative errors $(\delta a)_{\mathrm{RMS}}$, $(\delta c\rho)_{\mathrm{RMS}}$ and arithmetic mean values of errors $\bar{\delta} = \dfrac{(\delta a)_{\mathrm{RMS}} + (\delta c\rho)_{\mathrm{RMS}}}{2}$ of measuring thermophysical properties a and $c\rho$, calculations were made using formulas (17) and (25) with optimal values $\gamma^a_{\mathrm{opt}} = 0.48$; $\gamma^{c\rho}_{\mathrm{opt}} = 0.81$; $r_{\mathrm{opt}} = 5.25$ mm, the results of which are shown in Fig. 5*a*.

Fig. 5*a* shows that when the duration τ_{p} of the heat pulse changes, the arithmetic mean value of the root mean square estimates of the relative errors takes minimum values for $\tau_{\mathrm{p}}^{\mathrm{opt}} \approx 21$ s, in the range $18 < \tau_{\mathrm{p}} < 24$ s.

From dependence in Fig. 5*a* the reader might have the wrong impression that when the influence of the duration τ_{p} of the thermal pulse is taken into account, the measurement errors decrease by only 0.20–0.25 %. In fact, the application of the measurement method and the data processing technique proposed in the article allows reducing the arithmetic mean value of the mean square estimates of the relative errors $\bar{\delta} = \dfrac{(\delta a)_{\mathrm{RMS}} + (\delta c\rho)_{\mathrm{RMS}}}{2}$ by 10–20 % in comparison with the traditional method of linear "instantaneous" heat source [1–5, 7, 22].

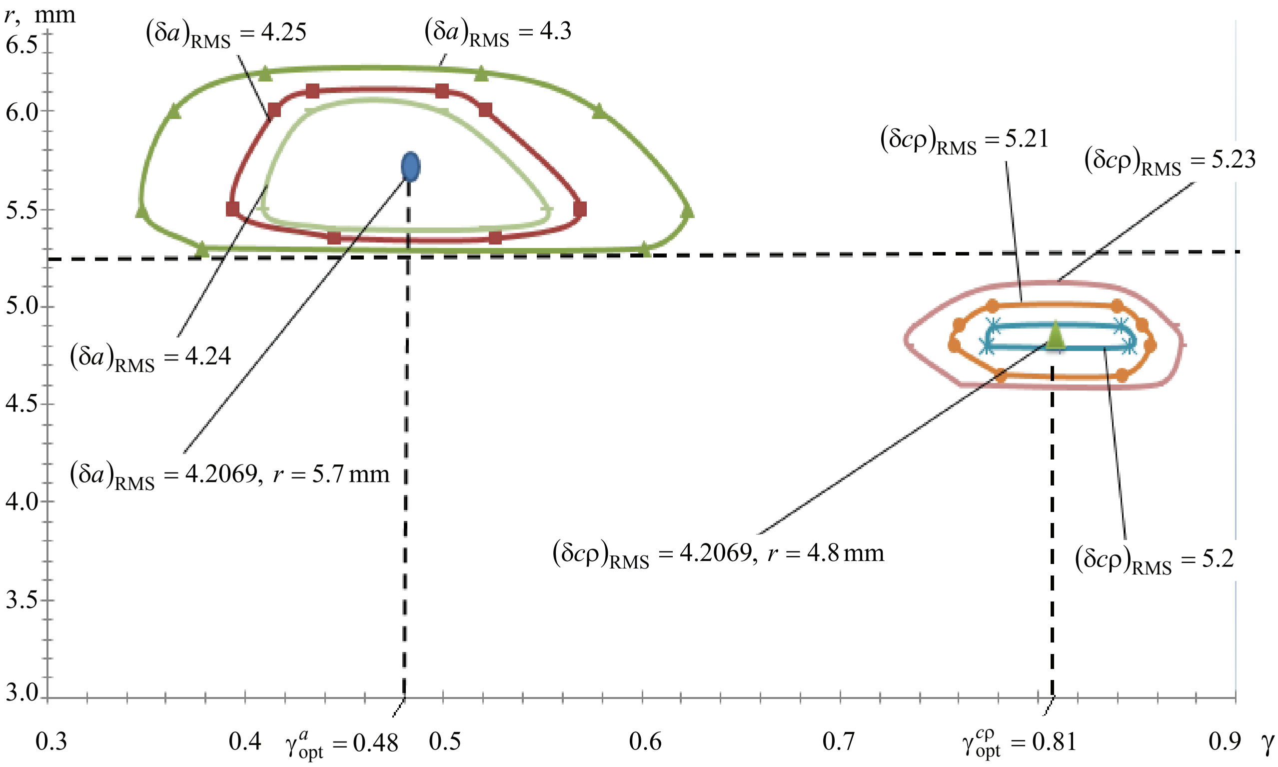

Fig. 4. Lines of equal levels of root-mean-square relative errors $(\delta a)_{RMS} = f_a(\gamma, r)$ and $(\delta c\rho)_{RMS} = f_{c\rho}(\gamma, r)$, constructed for optimal duration $\tau_p = 21$ s of the heat pulse

To illustrate this fact, we performed calculations of the coefficient of thermal diffusivity a and volumetric heat capacity $c\rho$ for various values of the duration τ_p of the heat pulse using:

– the calculated ratios proposed in this article (12) and (13);

– calculated ratios [1–5, 7, 22]

$$a_{inst} = \frac{r^2}{4\tau_{max}}; \quad c\rho_{inst} = \frac{Q_{lin}}{\pi\, e\, r^2 [T_{max} - T_0]}, \qquad (26)$$

used in the implementation of the traditional method of linear "instantaneous" heat source. The exact values were used in these calculations $a_{exact} = 1.06 \cdot 10^{-7}$ m²/s, $c\rho_{exact} = 1.83 \cdot 10^6$ J/(m³·K), $r_{opt} = 5.25$ mm, and the power value was calculated by the formula (23).

After calculating the values of a and $c\rho$ by formulas (12) and (13), as well as a_{inst} and $c\rho_{inst}$ by formula (26), errors $\delta a = \frac{a - a_{exact}}{a_{exact}} 100\,\%$, $\delta c\rho = \frac{c\rho - c\rho_{exact}}{c\rho_{exact}} 100\,\%$, $\delta a_{inst} = \frac{a_{inst} - a_{exact}}{a_{exact}} 100\,\%$, $\delta c\rho_{inst} = \frac{c\rho_{inst} - c\rho_{exact}}{c\rho_{exact}} 100\,\%$, and the arithmetic mean values $\bar{\delta} = \frac{[\delta a + \delta c\rho]}{2}$ and $\bar{\delta}_{inst} = \frac{[\delta a_{inst} + \delta c\rho_{inst}]}{2}$ were calculated.

As a result, dependency graphs $\bar{\delta} = f_1(\tau_p)$ и $\bar{\delta}_{inst} = f_2(\tau_p)$ presented in Fig. 5*b* were obtained.

As can be seen from the graphs shown in Fig. 5*b*, using the numerical modeling of measuring thermophysical properties, the following results were obtained:

1) when using the linear pulsed heat source method proposed in the article, the arithmetic average of the data processing errors $\bar{\delta} = f_1(\tau_p)$ do not exceed 1 %;

2) when processing data using the calculated relations (26), which underlie the traditional method of a linear "instantaneous" heat source [1–5, 7, 22], the arithmetic mean values of the data processing errors $\bar{\delta}_{inst} = \frac{[\delta a_{inst} + \delta c\rho_{inst}]}{2} = f_2(\tau_p)$ reach 14–25 % with a heat pulse duration in the range of $16 < \tau_p < 30$ s.

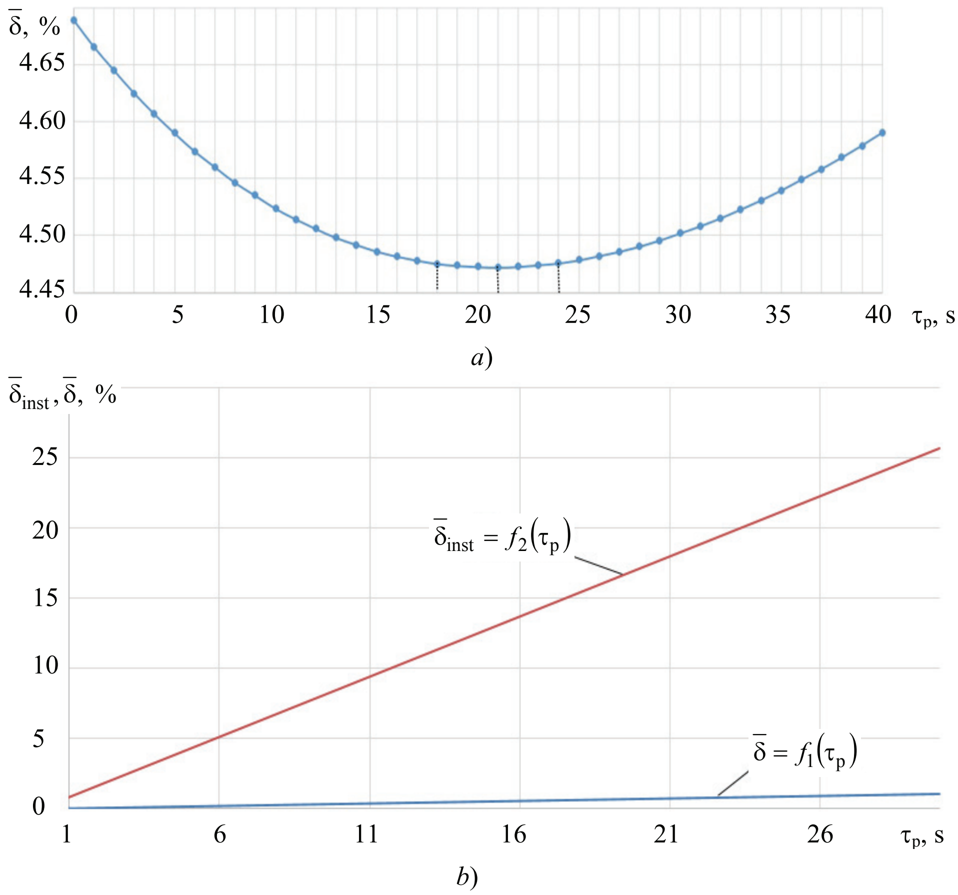

Fig. 5. Dependences of arithmetic mean values $\bar{\delta} = \dfrac{(\delta a)_{\mathrm{RMS}} + (\delta c\rho)_{\mathrm{RMS}}}{2}$ of mean square estimates of relative errors $(\delta a)_{\mathrm{RMS}}$, $(\delta c\rho)_{\mathrm{RMS}}$ of measuring thermal diffusivity a and volumetric heat capacity $c\rho$ on the duration τ_p of the heat pulse when processing data:

a – by the linear pulsed heat source method considered in this article;
b – traditional method of linear "instant" heat source [1–5, 7, 22]

The block diagram of the measuring installation is shown in Fig. 6. The thermoelectric converter *8*, in the form of a thermocouple, is connected to the normalizer *2*, amplifying and linearizing the signal of the thermocouple. Using the data acquisition board *3* of type E-14-140M manufactured by L-CARD (Russia), the thermal converter signal (emf) is measured, processed in a personal computer program *4* implemented in the LabView programming environment. The program also through a discrete output of the data acquisition board *3* controls the switch *7*, connecting the heater *9* to the power supply *6* for a given time. The obtained temperature response is processed in the program according to the method described above and the coefficients of thermal diffusivity and volumetric heat capacity of the studied material are calculated.

Experiments with the internal plant tissue of potatoes (Table 1) showed that the effective thermal conductivity of healthy tissue of the Udacha potato variety, as well as its volumetric heat capacity, are slightly different from the defective one, in particular, from tissue affected by dry rot, late blight, and alternariosis. The difference in the thermophysical properties of tissues can be explained by the different water content in them, as well as by a change in their structure.

The first block of Table 1 shows the arithmetic mean values of the coefficients of thermal conductivity and thermal diffusivity of the five measurement results for healthy plant tissue of potatoes. The boundaries of the confidence interval, taking into account the Student's coefficient, are ± 0.01 W/(m·K) and $\pm 0{,}15 \cdot 10^{-7}$ m^2/s, respectively, for the thermal

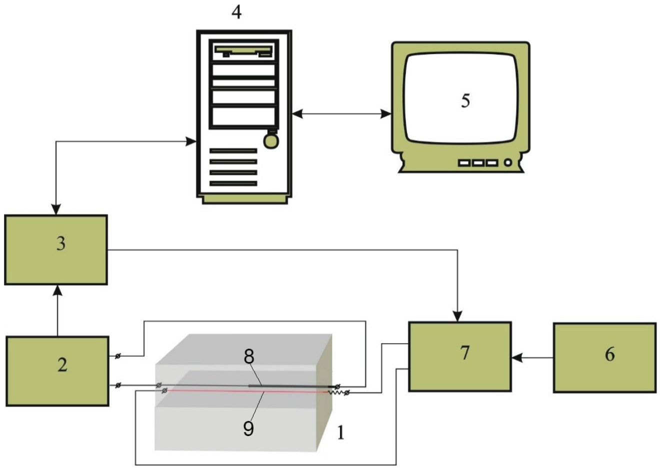

Fig. 6. Block diagram of the measuring installation:
1 – test material; *2* – normalizer; *3* – data acquisition board E14-140M; *4* – computer processing unit; *5* – visual display unit; *6* – power supply; *7* – switch; *8* – thermoelectric converter; *9* – wire linear heater

Potato tubers with internal plant tissue of different quality were used as the test material (Fig. 7).

Fig. 7. Placing a thermocouple and heater on the potato tuber plant tissue

Table 1

Thermophysical properties of healthy and defective internal plant tissue of potato

Test specimen	# measurement	λ, W/(m·K)	a, (×10^7), m^2/s	$c\rho$ (× 10^3), kJ/(m^3·K)
Healthy potato plant tissue	1	0.515	1.412	3.647
	2	0.510	1.383	3.684
	3	0.491	1.441	3.436
	4	0.505	1.421	3.554
	5	0.514	1.43	3.594
	Mean value	0.507	1.417	3.560
	rms deviation	0.010	0.022	0.011
Plant tissue of late blight potato	1	0.554	1.422	3.892
	2	0.542	1.462	3.707
	3	0.541	1.460	3.705
	4	0.561	1.360	4.125
	5	0.539	1.380	3.905
	Mean value	0.550	1.420	3.800
	rms deviation	0.009	0.046	0.017

conductivity and thermal diffusivity of the plant tissue, taking into account the Student's coefficient. The measured values in the sample were obtained for the same potato tuber.

The second block of Table 1 shows the arithmetic mean values of the coefficients of thermal conductivity and thermal diffusivity of the five measurement results for plant tissue of potato affected by late blight. The boundaries of the confidence interval, taking into account the Student's coefficient, are ± 0.025 W/(m·K) and $\pm 0.15 \cdot 10^{-7}$ m^2/s, respectively, for the thermal conductivity and thermal diffusivity of the plant tissue, taking into account the Student's coefficient. The measured values in the sample were obtained for the same potato tuber, but at different probe positions.

The time of the active stage of measuring the thermal conductivity and thermal diffusivity coefficients, as well as the specific volumetric heat capacity for one test sample does not exceed 1 min, and the product is heated less than 15 K, which fully meets the requirements for preserving the original properties of the test sample.

Recommendations for the implementation of the linear pulsed heat source method proposed in the article

It is noteworthy that when measuring the thermophysical properties of the test material, the thermal diffusivity *a* and volumetric heat capacity *c*ρ of which differ from the set values (in the initial data of the above calculations), one should do as follows:

1) by conducting preliminary measurements, it is necessary to determine the approximate values of the thermal diffusivity coefficient a_{app} and volumetric heat capacity $c\rho_{\text{app}}$ of the test material;

2) acting similar to the above mentioned in this article, it is necessary to:

a) carry out calculations (for the found values of a_{app} and $c\rho_{\text{app}}$) in order to determine (refine) the two optimal values of the parameter γ_{opt}^{a}, $\gamma_{\text{opt}}^{c\rho}$, as well as the structural dimensions r_{opt}^{a} and $r_{\text{opt}}^{c\rho}$ of the distance between the temperature meter and the linear heater;

б) take the distance between the temperature meter and the linear heater (the main structural size of the measuring device) equal to $r_{\text{opt}} = \dfrac{r_{\text{opt}}^{a} + r_{\text{opt}}^{c\rho}}{2}$, and calculate the value of the duration $\tau_{\text{p}}^{\text{opt}}$ of the thermal pulse in order to achieve $\overline{\delta} = \min$;

3) set the distance between the temperature meter and the linear heater in the measuring transducer $r_{\text{opt}} = \dfrac{r_{\text{opt}}^{a} + r_{\text{opt}}^{c\rho}}{2}$;

4) by carrying out a series of experiments (with manufactured samples), carry out measurements and subsequent processing of the obtained data (at found values of γ_{opt}^{a}, $\gamma_{\text{opt}}^{c\rho}$ and $\tau_{\text{p}} = \tau_{\text{p}}^{\text{opt}}$) and, as a result, obtain the values of the desired thermal diffusivity coefficient *a* and volumetric heat capacity *c*ρ of the test material.

Conclusion

Using the approach proposed in the article to the choice of two optimal values of the dimensionless parameter γ and the rational structural size *r*, which determines the relative position of the temperature meter and heater in the sample of the test material, provides a significant increase in the accuracy of measurements of the sought values of the thermal diffusivity *a* and volumetric heat capacity *c*ρ. Using the optimal value of the duration of the thermal pulse supplied to the linear heater, allows to further reduce the measurement error of the desired thermal physical properties.

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