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Mathematical Models of Temperature Fields of Potato Tubers with Surface and Internal Defects

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Abstract

Using the formulated assumptions and methods of the classical theory of heat conduction, mathematical models of the temperature fields of potato tubers in the form of boundary-value problems of heat conduction are developed. The cases of internal and surface defects in the form of mechanically damaged or diseased potato tissues are considered. It is assumed that flat, cylindrical or spherical defects in the tubers and on their surfaces are close to the configuration of the outer surfaces of the local sections of the tubers.

Along with three-dimensional mathematical models of temperature fields in potato tubers in general, mathematical models for local sections of tubers in the form of one-dimensional boundary-value heat conduction problems for multilayer systems, written in flat, cylindrical and spherical coordinate systems, have been developed for the first time.

The mathematical models presented in this article are intended for use in the design and development of an information measuring and control system for sorting potato tubers before storing them in order to increase the safety of potatoes during long-term storage.

Keywords

Potatoes; tubers; sorting; machine vision; thermal imaging methods; thermophysical properties; heat conduction; heat capacity; thermal diffusivity coefficient; temperature field; three-dimensional; one-dimensional; mathematical models; boundary value problems of heat conductivity.

Introduction

Preservation, transportation and storage of harvested potatoes are important tasks put forward before agro-industrial enterprises of the Russian Federation. Potato tubers must be carefully sorted out before going into the storage facility, and tubers that have been mechanically damaged or infected with phytophthora or rot must be removed to prevent infection of healthy tubers during storage. In this regard, the safety of potatoes, among other things, depends significantly on the probability of detecting defects before storage [1].

It is possible to detect the surface defects of tubers when humans examine them, but when sorting large amounts of potatoes, the probability of missing defective tubers is high due to inattention and fatigue. In addition, visual inspection does not allow detecting defects located under the surface of the object [1].

In recent years, thermal control systems are increasingly being used for the sorting of potato tubers. The principle of operation of such systems is the recognition of defects from controlled objects' images produced in the infrared region of the spectrum [1 – 3]. Such systems have advantages, since they provide the ability to detect both surface and subsurface defects [1, 4].

Defects of vegetables or fruits can be detected from their images produced in the infrared region of the spectrum in the presence of a temperature contrast

ΔT on the surfaces of healthy and damaged plant tissues. The object is exposed to a thermal effect to create ΔT in the process of monitoring. At the same time, the magnitude of the temperature contrast largely depends on the power and time of the thermal action, as well as on the thermophysical properties of the controlled object. For example, in [5] it was shown that ΔT reached the maximum value of 3.03 °C 65 ms after heating that lasted 10 ms. Two seconds after the end of heating, the value of ΔT reduced by 780 times compared to the maximum value, which did not allow detecting the defect. Thus, for objects that differ in physical properties, for example, potato tubers, the maximum values of ΔT for the same parameters of heat exposure will be different [1]. Therefore, it is important to solve the problems of determining the thermal parameters (intensity and duration) of the thermal effects on the potato tubers during the sorting process, under which the control system detects, with the required probability, the defects not only on the surface but also inside the tuber at some distance from its outer surface.

This paper aims to formulate possible variants of mathematical models of temperature fields of potato tubers in the form of boundary-value problems of heat conduction, given both surface and internal (subsurface) defects in tubers.

1. Assumptions taken when writing mathematical models

When writing the mathematical models formulated and presented below in the form of boundary-value heat conduction problems for calculation of temperature fields of potato tubers (or their local sections), the following assumptions were made.

1. The shape of defects (in the form of mechanical damage, dry or wet rot, or in the form of phytophthora, etc.) on the tubers is similar to that of the external surface of the tuber section, near which these defects are located.

2. On the surface of the tuber, there may be three types of areas:

– of flat shape,

– of cylindrical shape,

– of spherical shape,

near which there may be defects of the corresponding shape, i.e. flat, cylindrical and spherical.

3. The probability that the potato tuber has simultaneously all three types of surface defects is small, the presence of two types of defects is much higher, and most often there are tubers with defects of only one type (shape).

4. The initial temperature T_i of potato tubers for sorting can be considered constant and uniformly distributed throughout the potato tuber volume.

5. In the process of sorting, the potato tubers are heated with infrared radiation, and the temperature change T on the surface and inside the potato tuber does not exceed 5–7 °C.

6. With such a slight change in the temperature of the surface and tissues of the tuber, the thermophysical properties (both healthy tissues of potato tubers and defective parts) can be considered independent of temperature and constant in time.

7. In the process of sorting, potato tubers are exposed to the thermal effect in the form of infrared radiation pulse of constant power, and each tuber is exposed for a constant period ranging from three to six seconds.

8. Given that temperature of the infrared radiation source T_{ir} and the initial temperature T on the outer surface of potato tubers vary insignificantly during the sorting process and practically remain constant (i.e., $T_{ir} = \text{const}$, $T_i = \text{const}$), instead of the boundary condition prescribed by Stefan–Bolzmann law, the equation in the form of the relation [6]

$$q(\tau) = \sigma_S \left(T_{ir}^4 - T^4 \right),$$

where $q(\tau)$ is the heat flux supplied by the infrared radiation to the outer surface of the potato tuber over a known period of time τ_u; σ_S is reduced coefficient of radiation, one can use a simpler notation in the form of a boundary condition of the second kind

$$+\lambda \frac{\partial T(x,y,z,\tau)}{\partial n}\bigg|_{x,y,z \in S} = q(\tau) = \\ = \begin{cases} q = \text{const if } 0 < \tau < \tau_u; \\ 0 \text{ if } \tau \ge \tau_u, \end{cases}$$

where λ is heat conduction (of healthy or defective potato tuber tissue) in the area of its surface S; $x, y, z \in S$ are coordinates of x, y, z, belonging to the outer surface S of a potato tuber or a defect located on the surface of the tuber; a unit vector normal to the surface of the potato tuber or a defect located on the surface of the tuber; $q(\tau)$ is function that determines the change in heat flux in time; τ_u is duration of heat pulse $q = \text{const}$ in time τ.

9. In the process of sorting, it can be assumed that in the tissues of the potato tubers and in the defects, there are no internal sources of heat.

10. Heat transfer at the interfaces between healthy tuber tissues and defective tissues in the tubers can be

described by conventional boundary conditions of the 4th kind [6–8].

Taking into account the above assumptions, we have developed mathematical models of the temperature fields of potato tubers:

1) in the form of three-dimensional boundary value problems of heat conduction that determine the three-dimensional temperature fields of potato tubers as a whole, and the sections where flat, cylindrical and spherical defects are located;

a) at a certain depth near the surface of the tuber;

b) directly on the outer surface of the potato tuber;

2) in the form of one-dimensional boundary value problems of heat conduction determining one-dimensional temperature fields of local sections of tubers near flat, cylindrical and spherical defects located either at some depth from the outer surface of the tuber or directly on the outer surface of the potato tuber.

2. Mathematical models of temperature fields of potato tubers with flat, cylindrical and spherical defects located inside the tuber at a small distance from its surface

2.1. A physical model of potato tubers with flat, cylindrical and spherical defects located inside tubers at a small distance from the surface

Fig. 1 shows a physical model of a potato tuber with three possible types of defects located inside it. This figure shows that on the surface of the potato tuber there may be sections of:

– flat shape (in the upper part of the tuber);

– cylindrical shape (in the lower part of the tuber);

– spherical shape (in the left and right parts of the tuber)

Using the first assumption formulated above, it is most likely that defects on the potato tuber (in the form of dry or wet rot, or in the form of phytophthora disease, etc.) are similar to the shape of the surface section of the tuber near which the defects are located. Therefore, Fig. 1 shows that a flat defect in the shape of a plate I is most likely to occur near the flat upper part of the tuber. Similarly, a cylindrical defect II is likely to occur near the cylindrical lower part of the tuber, and a spherical defect III might occur in the local section of a spherical-shaped tuber.

Fig. 1 uses the following notation: x, y, z are the axes of the Cartesian coordinate system used in mathematical modeling of the potato tuber temperature field; r is the radial coordinates of the cylindrical and spherical coordinate systems used in the mathematical modeling of temperature fields (in the form of one-dimensional boundary value problems of heat conduction) on the local cylindrical II and spherical III sections of a potato tuber; V, V_1, V_2, V_3 are a set of internal points x, y, z belonging to healthy internal potato tissues V, internal tissues of flat defect I (V_1), inner tissues of cylindrical defect II (V_2), internal tissues of spherical defect III (V_3); S, S_1, S_2, S_3 are set of points x, y, z belonging to the outer surface of the healthy tissues of the potato tuber (S), the outer surface of the flat defect I tissue (S_1), the outer surface of the cylindrical defect II tissues (S_2), the outer surface of the spherical defect III tissues (S_3); T, λ, c, ρ, a are the

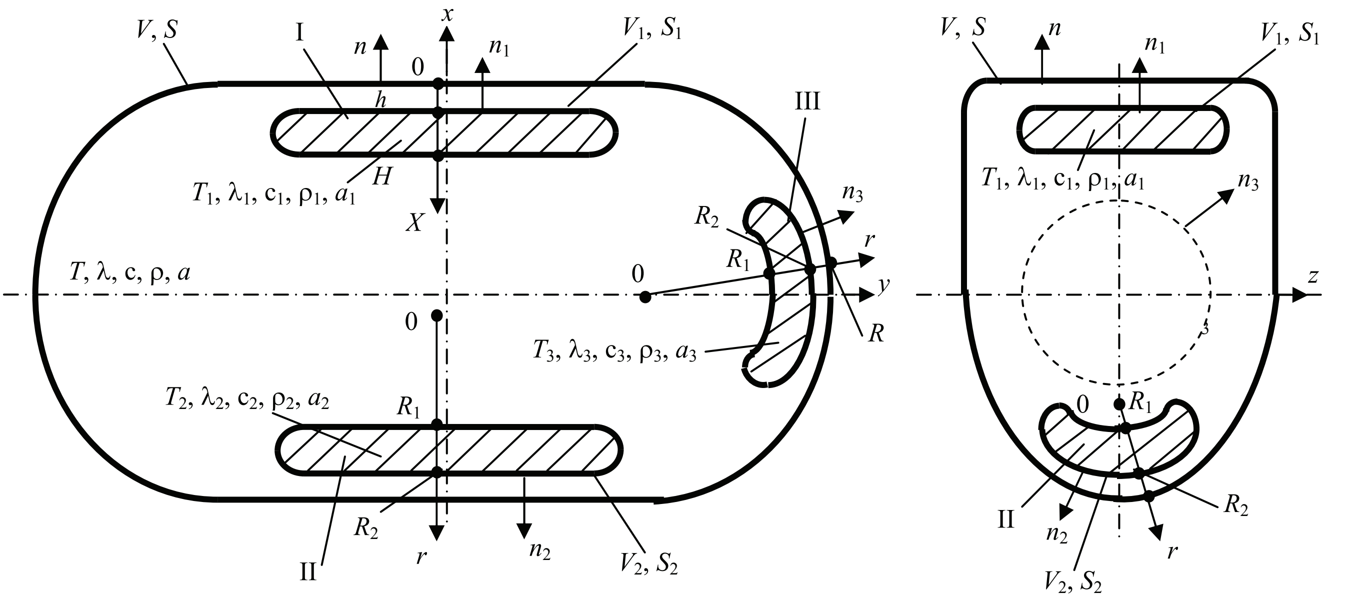

Fig. 1. Types of defects of flat (I), cylindrical (II) and spherical (III) shapes located inside the potato tuber

temperature, heat conduction, specific heat, density and thermal diffusivity coefficient of healthy potato tuber tissues, respectively; T_i, λ_i, c_i, ρ_i , a_i ($i = 1, 2, 3$) are specific heat, density and coefficient of thermal diffusivity of tissue defects I, II, III of potato tubers, respectively; n is a unit vector normal (perpendicular) to the surface S of the outer surface of the potato tuber; n_1, n_2, n_3 ($i = 1, 2, 3$) are the unit vectors normal (perpendicular) to the outer surfaces S_1, S_2, S_3 of the tissue defects I, II, III of potato tubers, respectively, X is the axis of the Cartesian coordinate system used for mathematical modeling of the temperature field (in the form of a one-dimensional boundary value heat conduction problem) on the local section of a flat defect I of a tuber; h, H are the values of the coordinates of the flat surface defect I on the X axis measured from the outer surface S of the tuber; r is radial axes of the coordinates of the cylindrical and spherical coordinate systems used in the mathematical modeling of temperature fields (in the form of one-dimensional boundary value problems of heat conduction) in the local cylindrical II and spherical III sections of potato tubers; R_1, R_2, R are the radial coordinates of the surfaces of defects II and III, and the outer surface of the potato tuber in cylindrical and spherical coordinate systems, respectively.

2.2. A mathematical model of the temperature field of a potato tuber with internal defects of all three shapes in the form of a three-dimensional boundary-value problem of heat conduction

A mathematical model of the temperature field of a potato tuber (containing all three types of defects inside) in the general case can be written in the form of the boundary-value problem of heat conduction:

$$c\rho \frac{\partial T(x,y,z,\tau)}{\partial \tau} = \operatorname{div}\left[\lambda \operatorname{grad} T(x,y,z,\tau)\right]; \quad \text{(B1)}$$

$$\tau > 0;\ x,y,z \in V \setminus (V_1 \cup V_2 \cup V_3);$$

$$c_1\rho_1 \frac{\partial T_1(x,y,z,\tau)}{\partial \tau} = \operatorname{div}\left[\lambda_1 \operatorname{grad} T_1(x,y,z,\tau)\right]; \quad \text{(B2)}$$

$$\tau > 0;\ x,y,z \in V_1;$$

$$c_2\rho_2 \frac{\partial T_2(x,y,z,\tau)}{\partial \tau} = \operatorname{div}\left[\lambda_2 \operatorname{grad} T_2(x,y,z,\tau)\right]; \quad \text{(B3)}$$

$$\tau > 0;\ x,y,z \in V_2;$$

$$c_3\rho_3 \frac{\partial T_3(x,y,z,\tau)}{\partial \tau} = \operatorname{div}\left[\lambda_3 \operatorname{grad} T_3(x,y,z,\tau)\right]; \quad \text{(B4)}$$

$$\tau > 0;\ x,y,z \in V_3;$$

$$T(x,y,z,0) = T_i = \text{const}; \quad \text{(B5-0)}$$

$$T_1(x,y,z,0) = T_i = \text{const}; \quad \text{(B5-1)}$$

$$T_2(x,y,z,0) = T_i = \text{const}; \quad \text{(B5-2)}$$

$$T_3(x,y,z,0) = T_i = \text{const}; \quad \text{(B5-3)}$$

$$+\lambda \frac{\partial T(x,y,z,\tau)}{\partial n}\bigg|_{x,y,z \in S} = q(\tau) = \begin{cases} q = \text{const if } 0 < \tau < \tau_u; \\ 0 \text{ if } \tau \ge \tau_u; \end{cases} \quad \text{(B6)}$$

$$T(x,y,z,\tau)\Big|_{x,y,z \in S_1} = T_1(x,y,z,\tau)\Big|_{x,y,z \in S_1}; \quad \text{(B7)}$$

$$\lambda \frac{\partial T(x,y.z.\tau)}{\partial n_1}\bigg|_{x,y,z \in S_1} = \lambda_1 \frac{\partial T_1(x,y.z.\tau)}{\partial n_1}\bigg|_{x,y,z \in S_1}; \quad \text{(B8)}$$

$$T(x,y,z,\tau)\Big|_{x,y,z \in S_2} = T_2(x,y,z,\tau)\Big|_{x,y,z \in S_2}; \quad \text{(B9)}$$

$$\lambda \frac{\partial T(x,y.z.\tau)}{\partial n_2}\bigg|_{x,y,z \in S_2} = \lambda_2 \frac{\partial T_2(x,y.z.\tau)}{\partial n_2}\bigg|_{x,y,z \in S_2}; \quad \text{(B10)}$$

$$T(x,y,z,\tau)\Big|_{x,y,z \in S_3} = T_3(x,y,z,\tau)\Big|_{x,y,z \in S_3}; \quad \text{(B11)}$$

$$\lambda \frac{\partial T(x,y,z,\tau)}{\partial n_3}\bigg|_{x,y,z \in S_3} = \lambda_3 \frac{\partial T_3(x,y,z,\tau)}{\partial n_3}\bigg|_{x,y,z \in S_3}; \quad \text{(B12)}$$

$$\frac{\partial T(x,y,z,\tau)}{\partial x}\bigg|_{x=0} = \frac{\partial T(x,y,z,\tau)}{\partial y}\bigg|_{y=0} = \frac{\partial T(x,y,z,\tau)}{\partial z}\bigg|_{z=0}, \quad \text{(B13)}$$

where $\operatorname{grad} T(x,y,z,\tau)$ is a differential operator that transforms the scalar temperature field $T(x,y,z,\tau)$ into a vector field; div is a differential operator that transforms a vector field $\lambda \operatorname{grad} T(x,y,z,\tau)$ into a scalar field; V, S are a set of points x, y, z inside and on the outer surface of the potato tuber; V_1, S_1 are a set of points x, y, z inside the flat defect I (Fig. 1), and on its outer surface; V_2, S_2 are a set of points x, y, z inside and on the outer surface of defect II of cylindrical shape; V_3, S_3 are a set of points x, y, z inside and on the outer surface of defect III of spherical shape; τ is time; T, λ, c, ρ, a are temperature, heat conduction, specific heat, density and coefficient of thermal diffusivity of healthy (non-defective) potato tuber tissue;

T_i, λ_i, c_i, ρ_i, a_i, $i = 1,2,3$ are temperature, heat conduction, specific heat, density and thermal diffusivity coefficient of defects I, II and III; n, n_1, n_2, n_3 are unit normal vectors directed in an outward direction relative to the surfaces S, S_1, S_2, and S_3; $x, y, z \in V$, $x, y, z \in Vi$ denote the interior points belonging to the sets V and V_i ($i = 1, 2, 3$). respectively; $x,\ y,\ z \in V \setminus \left(V_1 \cup V_2 \cup V_3\right)$ are sets of points obtained after subtraction of a union of sets $\left(V_1 \cup V_2 \cup V_3\right)$ form the set V; $x,\ y,\ z \in S$, $x,\ y,\ z \in Si$ are sets of points x, y, z, located on the surfaces S and S_i ($i = 1, 2, 3$); $T_i = \text{const}$ is initial temperature inside the potato tuber; q is heat flux; when denoting the equations of the boundary value problem of heat conduction (B1) – (B13), the letter B indicates the mathematical model of the temperature field of a potato tuber with internal defects located at a small distance from the surface.

When setting up the mathematical model written above, if there is no flat defect I in the tuber, it is necessary to remove the sets V_1, S_1 from the region of definition; this will lead to the need to eliminate the equation (B2), the initial condition (B5-1) and the equations of the boundary condition of the fourth kind (B7) and (B8) from the boundary value problem of heat conduction (B1) – (B13).

When setting the boundary value problem of heat conduction, if there is no cylindrical defect II in the tuber, it is necessary to remove the sets V_2, S_2 from the region of definition; this will lead to the necessity of eliminating the equation (B3), the initial condition (B5-2) and the equations of the boundary condition of the fourth kind (B9) and (B10) from the boundary value problem of heat conduction (B1) – (B13).

If there is no spherical defect III, the sets V_3 and S_3 should be removed from the region of definition, which leads to the need to eliminate the equation (B4), the initial condition (B5-3) and the fourth kind boundary condition in the form of equations (B11) and (B12).

The above-considered mathematical model of the temperature field of potato tubers with defects I, II and III located inside the tuber, written in the form of a boundary-value problem of heat conduction (B1) – (B13), is used to set and solve problems of calculating the temperature fields of potato tubers using the “TSTU” software package.

The mathematical model (B1) – (B13) can be substantially simplified if we consider the temperature fields only in that local part of the potato tuber where defects I, II or III are located.

2.2.1. A simplified mathematical model of the temperature field of the local part of a tuber near a flat-shaped defect I

On the local section of the potato tuber, inside which a flat-shaped defect I is located, we introduce the axis X of the Cartesian coordinate system. The beginning (point 0) of this axis is placed (Fig. 1) on the outer surface of the tuber, and the axis itself is directed into the potato tuber as shown in the upper part of Fig. 1. The distances from the surface of the tuber to the upper and lower parts of defect I are denoted by h and H, respectively. Then the simplified mathematical model of the temperature field in the local section of the tuber near the defect I can be written as a one-dimensional boundary-value problem of heat conduction:

$$\frac{\partial T\left(X,\tau\right)}{\partial T} = a\frac{\partial^2 T\left(X,\tau\right)}{\partial x^2},\ \ a = \frac{\lambda}{c\rho}, \tag{BP1}$$

$$\tau > 0,\ \ 0 < X < h,\ \ H < X < \infty;$$

$$\frac{\partial T_1\left(X,\tau\right)}{\partial T} = a_1\frac{\partial^2 T\left(X,\tau\right)}{\partial x^2},\ a_1 = \frac{\lambda_1}{c_1\rho_1},\ \tau > 0,\ h < X < H; \tag{BP2}$$

$$T\left(X,0\right) = T_1\left(X,0\right) = T_i = \text{const}; \tag{BP3}$$

$$-\lambda\frac{\partial T\left(0,\tau\right)}{\partial\tau} = q\left(\tau\right) = \begin{cases} q = \text{const if } 0 \le \tau \le \tau_u; \\ 0 \text{ if } \tau > \tau_u; \end{cases} \tag{BP4}$$

$$T\left(h,\tau\right) = T_1\left(h,\tau\right); \tag{BP5}$$

$$\lambda\frac{\partial T\left(h,\tau\right)}{\partial X} = \lambda_1\frac{\partial T_1\left(h,\tau\right)}{\partial X}; \tag{BP6}$$

$$T\left(H,\tau\right) = T_1\left(H,\tau\right); \tag{BP7}$$

$$\lambda_1\frac{\partial T_1\left(H,\tau\right)}{\partial X} = \lambda\frac{\partial T_1\left(H,\tau\right)}{\partial X}; \tag{BP8}$$

$$T\left(\infty,0\right) = T_i = \text{const}. \tag{BP9}$$

In the designation of the equations of the boundary-value problem of heat conduction (BP1) – (BP9), the letters BP indicate a mathematical model of the temperature field for a local section of a potato tuber with an internal flat-shaped defect located at a small distance from the surface.

2.2.2. A simplified mathematical model of the temperature field of the local part of a tuber near the cylindrical defect II

On the local section of the potato tuber, inside which there is a cylindrical defect II, we introduce the radial axis r of the cylindrical coordinate system. The beginning of this axis r is placed (Fig. 1) inside the tuber, and the axis r is directed towards the surface of the tuber. The distances from the origin of the r axis to the inner and outer surfaces of the defect II are denoted as R_1 and R_2, respectively, and the distance to the outer surface of the tuber is denoted by R. In this case, a simplified mathematical model of the temperature field of the local part of the tuber near the cylindrical defect II can be written in the form of a one-dimensional boundary-value problem of heat conduction:

$$\frac{\partial T(r,\tau)}{\partial \tau} = a\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T(r,\tau)}{\partial r}\right],\ a=\frac{\lambda}{c\rho}, \quad \text{(BZ1)}$$

$$\tau>0,\ 0<r<R_1,\ R_2<r<R;$$

$$\frac{\partial T_2(r,\tau)}{\partial \tau} = a_2\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T(r,\tau)}{\partial r}\right],\ a_2=\frac{\lambda_2}{c_2\rho_2}, \quad \text{(BZ2)}$$

$$\tau>0,\ \ R_1<r<R_2;$$

$$T(r,0)=T_2(r,0)=T_i=\text{const}; \quad \text{(BZ3)}$$

$$T(0,\tau)=T_i=\text{const}; \quad \text{(BZ4)}$$

$$T(R_1,\tau)=T_2(R_1,\tau); \quad \text{(BZ5)}$$

$$\lambda\frac{\partial T(R_1,\tau)}{\partial r}=\lambda_2\frac{\partial T_2(R_1,\tau)}{\partial r}; \quad \text{(BZ6)}$$

$$T_2(R_2,\tau)=T(R_2,\tau); \quad \text{(BZ7)}$$

$$\lambda_2\frac{\partial T_2(R_2,\tau)}{\partial r}=\lambda\frac{\partial T(R,\tau)}{\partial r}; \quad \text{(BZ8)}$$

$$\lambda\frac{\partial T(R,\tau)}{\partial \tau}=q(\tau)=\begin{cases} q=\text{const if } 0\le\tau\le\tau_u; \\ 0 \text{ if } \tau>\tau_u. \end{cases} \quad \text{(BZ9)}$$

In the designation of the equations for the boundary-value problem of heat conduction (BZ1) – (BZ9), the letters of the BZ indicate that the mathematical model of the temperature field for a local section of a potato tuber with an internal cylindrical defect located at a small distance from the surface is considered.

2.2.3. A simplified mathematical model of the temperature field of a local part of a potato tuber near the spherical defect III

In that local part of the potato tuber, inside which the spherical defect III is located, we introduce the radial axis r of the spherical coordinate system. The origin of the r axis is placed (Fig. 1) inside the tuber, and the axis r is directed towards the surface of the tuber. The distances from the origin of the r axis to the inner and outer spherical surfaces of defect III are denoted as R_1 and R_2, respectively, and the distance to the spherical surface of the tuber (located above the defect III) is denoted as R. Then a simplified mathematical model of the temperature field of the local part of the potato tuber near the spherical defect III is represented in the form of a boundary-value problem of heat conduction:

$$\frac{\partial T(r,\tau)}{\partial r} = a\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial T(r,\tau)}{\partial r}\right],\ a=\frac{\lambda}{c\rho}, \quad \text{(BC1)}$$

$$\tau>0,\ 0<r<R_1,\ R_2<r<R;$$

$$\frac{\partial T_3(r,\tau)}{\partial \tau} = a_3\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial T(r,\tau)}{\partial r}\right],\ a_3=\frac{\lambda_3}{c_3\rho_3}, \quad \text{(BC2)}$$

$$\tau>0,\ \ R_1<r<R_2;$$

$$T(r,0)=T_3(r,0)=T_i=\text{const}; \quad \text{(BC3)}$$

$$T(0,\tau)=T_i=\text{const}; \quad \text{(BC4)}$$

$$T(R_1,\tau)=T_3(R_1,\tau); \quad \text{(BC5)}$$

$$\lambda\frac{\partial T(R_1,\tau)}{\partial r}=\lambda_3\frac{\partial T_3(R_1,\tau)}{\partial r}; \quad \text{(BC6)}$$

$$T_3(R_2,\tau)=T(R_2,\tau); \quad \text{(BC7)}$$

$$\lambda_3\frac{\partial T_3(R_2,\tau)}{\partial r}=\lambda\frac{\partial T(R_3,\tau)}{\partial r}; \quad \text{(BC8)}$$

$$\lambda\frac{\partial T(R,\tau)}{\partial \tau}=q(\tau)=\begin{cases} q=\text{const if } 0\le\tau\le\tau_u, \\ 0 \text{ if } \tau>\tau_u. \end{cases} \quad \text{(BC9)}$$

In the designation of the equations of the boundary value problem of heat conduction (BC1) – (BC9), the letters BC indicate that the mathematical model of the temperature field for a local part of a potato tuber with an inner spherical defect located at a small distance from the surface is considered.

The one-dimensional mathematical models of temperature fields presented above in the local sections (parts) of potato tubers in the form of boundary-value problems of heat conduction (BP1) – (BP9), (BZ1) – (BZ9), (BC1) – (BC9) make it possible to calculate the temperature fields using simplified algorithms in comparison with the general case of complex three-dimensional mathematical model (B1) – (B13) considered above in 2.2. The use of simpler algorithms opens up great possibilities for reducing the computer time spent on computing.

3. Mathematical models of temperature fields of potato tubers with flat, cylindrical and spherical defects located directly on the surface of tubers

3.1. A physical model of potato tubers with flat, cylindrical and spherical defects located on the surface of tubers

Fig. 2 shows a physical model of a potato tuber with three possible types of defects located on its surface. On the surface of the potato tuber, there may be the following areas:

– of flat shape (at the top of the tuber);

– of cylindrical shape (at the bottom of the tuber);

– of spherical shape (in the left and right parts of the tuber).

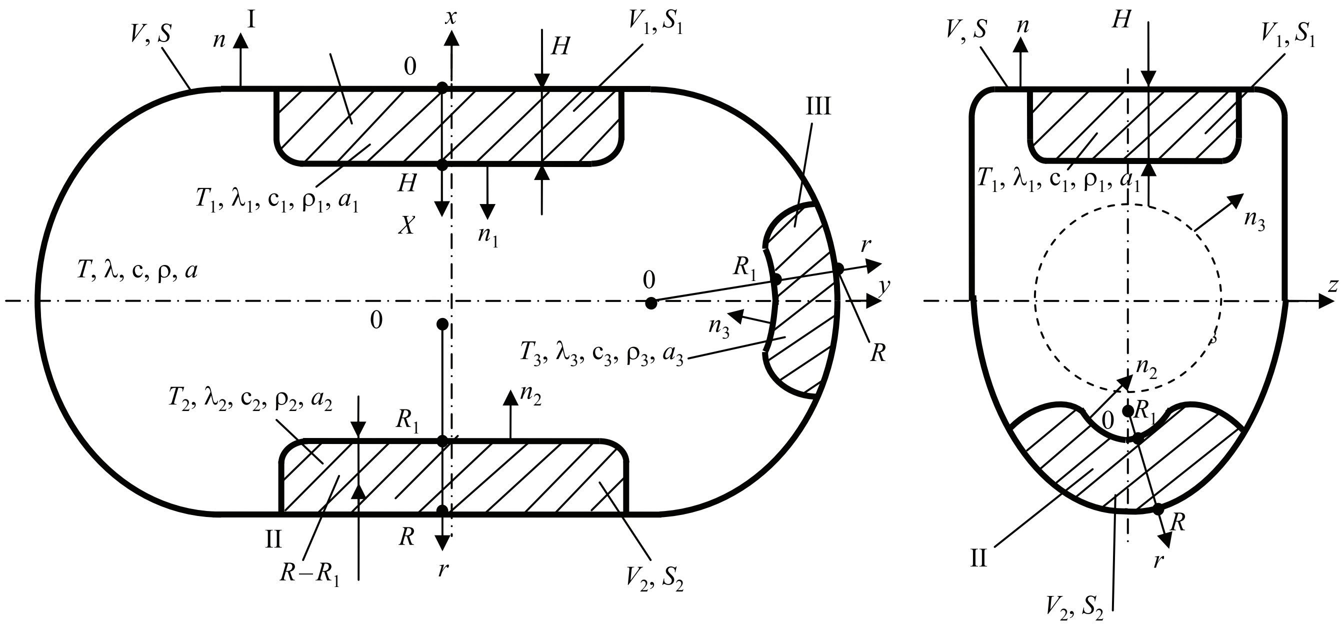

Fig. 2. Types of defects of flat (I), cylindrical (II) and spherical (III) shapes located on the surface of the potato tuber

On the basis of the first assumption formulated in paragraph 1 above, it is most likely that the defects on the potato tuber (in the form of dry or wet rot, or in the form of phytophthora, etc.) are similar in shape to the surface area of the tuber, where the defects are located. Therefore. Fig. 2 shows that on the flat upper part of the tuber the flat-shaped defect I is most likely to occur. Likewise, on the surface of the cylindrical lower part of the tuber a defect of the cylindrical shape II will occur with a high probability, and in the local area of a tuber of spherical form, a defect of spherical shape III can occur.

Fig. 2 uses basically the same notation as Fig. 1. The main difference is that defects of potato tubers shown in Fig. 2 are depicted directly on the surfaces of the tuber. In this regard, to determine the geometric coordinate of the interface between the healthy tissue of the tuber and the diseased tissue with the flat-shaped defect I (on the X axis of the Cartesian coordinate system) we only one value of the thickness H. Similarly, when determining the geometric coordinates of the interface between a healthy tuber tissue and a diseased (unhealthy, damaged) tissue with cylindrical defects II and spherical defects III (on the radial axes r of the cylindrical and spherical coordinate systems), it was required to show only one value of the distance R_1 from the origin $r = 0$ to the lower boundary of the corresponding defects. In this case, to identify the upper boundaries of defects II and III, which coincide with the outer surface of the tuber, it was enough to use one common designation R.

3.2. A mathematical model of the temperature field of a potato tuber with defects of all three shapes located on its outer surface in the form of a three-dimensional boundary-value problem of heat conduction

A mathematical model of the temperature field of a potato tuber, on the outer surface of which there are three types of defects, can be written in the form of the boundary value problem of heat conduction:

$$c\rho \frac{\partial T(x,y,z,\tau)}{\partial \tau} = \mathrm{div}\left[\lambda \cdot \mathrm{grad}\, T(x,y,z,\tau)\right]; \quad \text{(H1)}$$

$$\tau > 0; \quad x,y,z \in V \setminus (V_1 \cup V_2 \cup V_3);$$

$$c_1\rho_1 \frac{\partial T_1(x,y,z,\tau)}{\partial \tau} = \mathrm{div}\left[\lambda_1 \cdot \mathrm{grad}\, T_1(x,y,z,\tau)\right]; \quad \text{(H2)}$$

$$\tau > 0; \quad x,y,z \in V_1;$$

$$c_2\rho_2 \frac{\partial T_2(x,y,z,\tau)}{\partial \tau} = \mathrm{div}\left[\lambda_2 \cdot \mathrm{grad}\, T_2(x,y,z,\tau)\right]; \quad \text{(H3)}$$

$$\tau > 0; \quad x,y,z \in V_2;$$

$$c_3\rho_3 \frac{\partial T_3(x,y,z,\tau)}{\partial \tau} = \mathrm{div}\left[\lambda_3 \cdot \mathrm{grad}\, T_3(x,y,z,\tau)\right]; \quad \text{(H4)}$$

$$\tau > 0; \quad x, y, z \in V_3;$$

$$T(x,y,z,0) = T_i = \mathrm{const}; \quad \text{(H5-0)}$$

$$T_1(x,y,z,0) = T_i = \mathrm{const}; \quad \text{(H5-1)}$$

$$T_2(x,y,z,0) = T_i = \mathrm{const}; \quad \text{(H5-2)}$$

$$T_3(x,y,z,0) = T_i = \mathrm{const}; \quad \text{(H5-3)}$$

$$+\lambda \frac{\partial T(x,y,z,\tau)}{\partial n}\bigg|_{x,y,z \in S \setminus (S_1 \cup S_2 \cup S_3)} = q(\tau); \tag{H6-0}$$

$$+\lambda_1 \frac{\partial T_1(x,y,z,\tau)}{\partial n_1}\bigg|_{x,y,z \in S_1 \cap S} = q(\tau); \tag{H6-1}$$

$$+\lambda_2 \frac{\partial T_2(x,y,z,\tau)}{\partial n_2}\bigg|_{x,y,z \in S_2 \cap S} = q(\tau); \tag{H6-2}$$

$$+\lambda_3 \frac{\partial T_3(x,y,z,\tau)}{\partial n_3}\bigg|_{x,y,z \in S_3 \cap S} = q(\tau); \tag{H6-3}$$

$$q(\tau) = \begin{cases} q = \text{const if } 0 < \tau < \tau_u; \\ 0 \text{ if } \tau_u \le \tau < \infty; \end{cases}$$

$$T(x,y,z,\tau)\Big|_{x,y,z \in S_1 \setminus (S_1 \cap S)} = T_1(x,y,z,\tau)\Big|_{x,y,z \in S_1 \setminus (S_1 \cap S)}; \tag{H7}$$

$$\lambda \frac{\partial T(x,y.z.\tau)}{\partial n_1}\bigg|_{x,y,z \in S_1 \setminus (S_1 \cap S)} = \lambda_1 \frac{\partial T_1(x,y.z.\tau)}{\partial n_1}\bigg|_{x,y,z \in S_1 \setminus (S_1 \cap S)}; \tag{H8}$$

$$T(x,y,z,\tau)\Big|_{x,y,z \in S_2 \setminus (S_2 \cap S)} = T_2(x,y,z,\tau)\Big|_{x,y,z \in S_2 \setminus (S_2 \cap S)}; \tag{H9}$$

$$\lambda \frac{\partial T(x,y.z.\tau)}{\partial n_2}\bigg|_{x,y,z \in S_2 \setminus S_2 \cap S} = \lambda_2 \frac{\partial T_2(x,y.z.\tau)}{\partial n_2}\bigg|_{x,y,z \in S_2 \setminus (S_2 \cap S)}; \tag{H10}$$

$$T(x,y,z,\tau)\Big|_{x,y,z \in S_3 \setminus (S_3 \cap S)} = T_3(x,y,z,\tau)\Big|_{x,y,z \in S_3 \setminus (S_3 \cap S)}; \tag{H11}$$

$$\lambda \frac{\partial T(x,y,z,\tau)}{\partial n_3}\bigg|_{x,y,z \in S_3 \setminus (S_3 \cap S)} = \lambda_3 \frac{\partial T_3(x,y,z,\tau)}{\partial n_3}\bigg|_{x,y,z \in S_3 \setminus (S_3 \cap S)}; \tag{H12}$$

$$\frac{\partial T(x,y,z,\tau)}{\partial x}\bigg|_{x=0} = \frac{\partial T(x,y,z,\tau)}{\partial y}\bigg|_{y=0} = \frac{\partial T(x,y,z,\tau)}{\partial z}\bigg|_{z=0} = 0, \tag{H13}$$

When writing the mathematical model (H1) – (H13), basically the same notation was used as in writing the mathematical model (B1) – (B13) considered above in 2.2. However, in connection with the specificity of the boundary-value problem of heat conduction (H1) – (H13), there is one insignificant difference, which consists in using $x, y, z \in S \setminus (S_1 \cup S_2 \cup S_3)$, denoting a set of points x, y, z, obtained after subtraction of the union of sets $(S_1 \cup S_2 \cup S_3)$ from the set S. In the designation of the equations of the boundary-value problem of heat conduction (H1) – (H13), the letter H indicates that the mathematical model of the temperature field of a potato tuber with external defects located directly on the surface of the tuber is considered.

When writing the mathematical model above, if a flat-shaped defect I is missing in the tuber, sets V_1, S_1 should be removed from the region of definition; this leads to the need to eliminate the equation (H2), the initial condition (H5-1), the boundary condition (H6-1) and the equations of the boundary condition of the fourth kind (H7) and (H8) from the boundary value problem of heat conductivity (H1) – (H13).

When setting the boundary value problem of heat conduction, if a cylindrical defect II is missing in the tuber, the sets V_2, S_2 should be removed from the region of definition; this leads to the need to eliminate the equation (H3), the initial condition (H5-2), the boundary condition (H6-2) and the equations of the boundary condition of the fourth kind (H9) and (H10) from the boundary value problem of heat conductivity (H1) – (H13).

If a spherical defect III is missing, the sets V_3 and S_3 should be removed from the region of definition, which leads to the need to eliminate the equation (H4), the initial condition (H5-3), the boundary condition (H6-3) and the fourth kind boundary condition in the form of equations (H11) and (H12) from the boundary value problem of heat conductivity (H1) – (H13).

The above-mentioned mathematical model of the temperature field of a potato tuber with defects I, II and III located directly on the surface of the tuber, written in the form of the boundary-value problem of heat conduction (H1) – (H13), is used to set and solve problems of calculating the temperature fields of potato tubers using the TSTU software package.

The mathematical model (H1) – (H13) can be substantially simplified if we consider the temperature fields only in that local area of the potato tuber where defects I, II or III are located.

3.2.1. A simplified mathematical model of the temperature field of the local section of a potato tuber near the surface of a flat-shaped defect I

We consider a simplified mathematical model of the potato temperature field in the section of a flat area of its surface if the defect I (Fig. 2) is located directly on the surface of the tuber and has the shape of a plate of thickness H. In this local section of the tuber, we introduce the *X* axis, the origin of which is placed on the outer surface of the defect, and the *X* axis itself is directed into the potato tuber.

In this case, a one-dimensional mathematical model of the temperature field near a local defective part of the tuber surface having a flat-shaped defect I is written in the form of a one-dimensional boundary-value problem of heat conduction:

$$\frac{\partial T_1(X,\tau)}{\partial \tau} = a_1 \frac{\partial^2 T_1(X,\tau)}{\partial x^2},\ a_1 = \frac{\lambda_1}{c_1 \rho_1},\ \tau > 0,\ 0 < X < H; \qquad \text{(HP1)}$$

$$\frac{\partial T(X,\tau)}{\partial \tau} = a \frac{\partial^2 T(X,\tau)}{\partial x^2},\ a = \frac{\lambda}{c\rho},\ \tau > 0,\ H < X < \infty; \qquad \text{(HP2)}$$

$$T_1(X,0) = T(X,0) = T_i = \text{const}; \qquad \text{(HP3)}$$

$$-\lambda_1 \frac{\partial T_1(0,\tau)}{\partial x} = q(\tau) = \begin{cases} q = \text{const if } 0 < \tau \le \tau_u; \\ 0 \text{ if } \tau \ge \tau_u; \end{cases} \qquad \text{(HP4)}$$

$$T_1(H,\tau) = T(H,\tau); \qquad \text{(HP5)}$$

$$\lambda_1 \frac{\partial T_1(H,\tau)}{\partial x} = \lambda \frac{\partial T(H,\tau)}{\partial x}; \qquad \text{(HP6)}$$

$$T(\infty,\tau) = T_i = \text{const}. \qquad \text{(HP7)}$$

In the designation of the equations of the boundary-value problem of heat conduction (HP1) – (HP7), the letters HP indicate that a mathematical model of the temperature field for a local section of a potato tuber with an external flat defect located directly on the surface of the tuber is considered.

3.2.2. A mathematical model of the temperature field in the local section of the potato tuber with a cylindrical defect II on the surface

We consider a simplified mathematical model of the temperature field of a potato tuber in the section of its surface with a cylindrical defect II (Fig. 2). In this part of the tuber, we introduce the radial axis r of the cylindrical coordinate system, the origin of this axis is placed inside the tuber, and we direct this axis towards the surface of the tuber (Fig. 2) in such a way that the inner surface of defect II corresponds to the radial coordinate *r* R_1, and the surface of defect II (coinciding with the outer surface of the potato tuber) corresponds to the value of the radial coordinate $r = R$.

In view of the foregoing, a simplified mathematical model of the temperature field in the section of the tuber surface of a cylindrical shape with an outer radius *R* (with a cylindrical defect III with an internal radius R_1 located on this surface area) is written as a one-dimensional boundary-value problem of heat conduction:

$$\frac{\partial T_2(r,\tau)}{\partial \tau} = a_2 \frac{1}{r}\frac{\partial}{\partial r}\left[r \frac{\partial T(r,\tau)}{\partial r} \right],\ a_2 = \frac{\lambda_2}{c_2 \rho_2}, \qquad \text{(HZ1)}$$
$$\tau > 0,\ 0 < r < R_1;$$

$$\frac{\partial T(r,\tau)}{\partial r} = a \frac{1}{r}\frac{\partial}{\partial r}\left[r \frac{\partial T(r,\tau)}{\partial r} \right],\ a = \frac{\lambda}{c\rho}, \qquad \text{(HZ2)}$$
$$\tau > 0,\ R_1 < r < R;$$

$$T_2(r,0) = T(r,0) = T_i = \text{const}; \qquad \text{(HZ3)}$$

$$+\lambda_2 \frac{\partial T_2(R,\tau)}{\partial \tau} = q(\tau) = \begin{cases} q = \text{const if } 0 \le \tau \le \tau_u; \\ 0 \text{ if } \tau > \tau_u; \end{cases} \qquad \text{(HZ4)}$$

$$T_2(R_1,\tau) = T(R_1,\tau); \qquad \text{(HZ5)}$$

$$\lambda_2 \frac{\partial T_2(R_1,\tau)}{\partial r} = \lambda \frac{\partial T(R_1,\tau)}{\partial r}; \qquad \text{(HZ6)}$$

$$T(0,\tau) = T_i = \text{const}. \qquad \text{(HZ7)}$$

In the designation of the equations for the boundary-value problem of heat conduction (HZ1) – (HZ7), the letters HZ indicate that a mathematical model of the temperature field for a local section of a potato tuber with an external cylindrical defect located directly on the surface of the tuber is considered.

3.2.3. A mathematical model of the temperature field in the local section of the potato tuber with a spherical defect III on the surface

We consider a simplified mathematical model of the temperature field of a potato tuber in the region of its spherical surface area with an outer radius *R*, with a spherical defect III (Fig. 2) located on the tuber surface and with an internal radius R_1. In this section of the tuber, we introduce the radial axis *r* of the spherical coordinate system, with the origin of the axis located inside the tuber, and direct this axis towards the outer surface of the tuber so that the inner and outer surfaces of the spherical defect III have radial coordinates $r = R_1$ and $r = R$.

Then a simplified mathematical model of the temperature field for the section of the tuber surface with a spherical defect is written in the form of a one-dimensional boundary-value problem of heat conduction:

$$\frac{\partial T_3(r,\tau)}{\partial \tau} = a\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T_3(r,\tau)}{\partial r}\right],\ a_3 = \frac{\lambda_3}{c_3\rho_3}, \quad \text{(HC1)}$$
$$\tau > 0,\quad R_1 < r < R;$$

$$\frac{\partial T(r,\tau)}{\partial \tau} = a\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T(r,\tau)}{\partial r}\right],\ a = \frac{\lambda}{c\rho}, \quad \text{(HC2)}$$
$$\tau > 0,\quad 0 < r < R_1;$$

$$T_3(r,0) = T(r,0) = T_i = \text{const}; \quad \text{(HC3)}$$

$$\lambda_3\frac{\partial T_3(R,\tau)}{\partial r} = q(\tau) = \begin{cases} q = \text{const if } 0 \le \tau \le \tau_u; \\ 0 \text{ if } \tau > \tau_u; \end{cases} \quad \text{(HC4)}$$

$$T_3(R_1,\tau) = T(R_1,\tau), \quad \text{(HC5)}$$

$$\lambda_3\frac{\partial T_3(R_1,\tau)}{\partial r} = \lambda\frac{\partial T(R_1,\tau)}{\partial r}; \quad \text{(HC6)}$$

$$T(0,\tau) = T_i = \text{const}. \quad \text{(HC7)}$$

In the designation of the equations for the boundary-value problem of heat conduction (HC1) – (HC7), the letters HC indicate that the mathematical model of the temperature field for a local section of a potato tuber with an external spherical defect located directly on the surface of the tuber is considered.

The above-described one-dimensional mathematical models of temperature fields in local sections of potato tubers (with defects located on the outer surface) in the form of boundary-value problems of heat conductivity (HP1) – (HP7), (HZ1) – (HZ7), (HC1) – (HC7) are used to calculate the temperature fields using simpler algorithms in comparison with the general case of using the complex three-dimensional mathematical model (H1) – (H13), considered above in 3.2. The use of simplified algorithms opens up great possibilities for reducing the time spent on computing.

For the practical application of the mathematical models formulated in the article, it is necessary to know the values of thermophysical properties (heat conduction, volume heat capacity and thermal diffusivity) that are introduced into the initial three-dimensional and simplified one-dimensional boundary value problems of heat conductivity. For experimental determination of these thermophysical properties, the methods and measuring devices given in [8–12] can be used. The questions of optimization of the process parameters, methods and the choice of rational structural dimensions of devices for measuring thermophysical properties are considered in [13–17].

Conclusion

The results of the research presented in the form of mathematical models of three-dimensional temperature fields of potato tubers in general, and one-dimensional temperature fields of tuber sections of flat, cylindrical and spherical shapes (with defects of the same shape contained therein) are used to solve problems of selecting optimal process parameters of thermal effects, rational design dimensions of infrared radiation receivers installed in the data-measuring and control system that is used to sort out potato tubers before storage.

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