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Optimization of the Heat Pulse Duration, Basic Construction Size of the Device and Conditions of Experimental Data Processing when Measuring Thermophysical Properties of Heat Insulating Materials by Pulse Plane Heat Source Method to Improve the Testing Laboratory Management System

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Abstract

Using the developed mathematical models of errors in measuring the heat conductivity and the thermal diffusivity coefficient of heat insulating materials, the authors propose a method for selecting the optimal: (1) value of the heat pulse duration; (2) basic construction size of the device, and 3) conditions for implementing the algorithm of the experimental data processing.

Keywords

Heat conductivity; thermal diffusivity coefficient; measurement; minimization of errors; optimal data processing conditions; rational value of construction size; heat pulse duration.

Introduction

The need for information on thermophysical properties of heat insulating materials arises while designing new technological processes, controlling product quality in conditions of real-life production processes, as well as while doing mathematical modeling and solving optimization problems of modernized industries [1 – 5]. The main approach to obtaining knowledge of thermophysical properties for new substances and materials remains their experimental measurement [1 – 8].

Traditional methodology of implementing methods of "instantaneous" heat sources did not pay due attention to the choice of [1 – 4]:

1) optimal conditions for measuring and processing primary information;

2) the rational constructionsizes of the used measuring devices, 3) the actual heat pulse duration τ_{pulse}.

Only recent publications [5 – 10] have discussed the optimization of the mode parameters in the measurement process and rational values of construction sizes of measuring devices. However the choice of the optimal heat pulse duration has not been considered.

The aim of the research results presented in the article is to increase the accuracy of measuring thermophysical properties of heat insulating materials by the pulse plane heat source method by selecting the optimal conditions for both the measurement process of thermophysical properties and the algorithm for processing the experimental data.

To achieve this goal, the following tasks have been set and solved:

1) a mathematical formulation of the problem of choosing the optimal conditions for carrying out the experiment and subsequent processing of the obtained experimental data was made for the pulse plane heat source method;

2) the problem of choosing the optimal: a) heat pulse duration; b) basic construction size of the measuring device; c) parameters of the algorithm for processing experimental data, has been solved;

3) recommendations on the implementation of the pulse plane heat source method when measuring thermophysical properties of heat insulating materials have been formulated.

A significant drawback of the known methods of a plane "instantaneous" heat source is that the mathematical model of the temperature field $T(x, \tau)$ in the sample is represented in the form [1 – 10]:

$$c\rho \frac{\partial T(x,\tau)}{\partial \tau} = \lambda \frac{\partial^2 T(x,\tau)}{\partial x^2} + W(x,\tau),$$

$$\tau > 0, \ -\infty < x < +\infty ;$$

$$T(x, 0) = T_0 = 0 ;$$

$$T(-\infty, \tau) = T(+\infty, \tau) = T_0 = 0 ,$$

where the internal heat source $W(x, \tau)$ is given as a plane instantaneous pulse $W(x,\tau) = Q_n \delta(x)\delta(\tau)$, but in fact the heat is supplied to the heater for a period of time $0 < \tau \le \tau_{\text{pulse}}$.

The designations used above: x, τ is the spatial coordinate of the sample and time; $c\rho$, λ is volumetric heat capacity and heat conductivity of the studied material; T_0 is the initial temperature of the material (at time $\tau = 0$) taken as the start of the temperature scale in each experiment, i.e. $T_0 = 0$; Q_n is the amount of heat released per unit surface of the flat heater at $x = 0$ and at time $\tau = 0$; $\delta(x)$, $\delta(\tau)$ – the symbolic Dirac delta functions [1, 5, 11, 12], τ_{pulse} is the duration of the real (not instantaneous) heat pulse applied to the heater.

The physical model of the measuring device is a cell (Fig. 1) where a sample which consists of three plates: the lower plate *2*, the middle plate *1* and the top plate *3* is placed. The highest requirements are imposed on the accuracy of manufacturing the middle plate 1 of a given thickness *x*, the upper and lower edges of which must be made strictly parallel to each other. The low-inertia flat heater *4* is usually placed between the lower plate *2* and the middle plate *1*, and the primary temperature measuring transducer, e.g. a thermocouple *5*, is installed between the middle and the top plates.

The measuring cell shown in Fig. 1 includes the following main elements.

I. The sample of the studied material made in the form of three plate elements *1*, *2*, *3*. Note that the thickness *x* of the plate element *1* along the *x* axis is selected within the range of 2 to 10 mm depending on the thermophysical properties of the studied heat insulating material. The heights L_2 and L_3 along the *x* axis of elements *2* and *3* of the studied heat insulating material should be about 60 mm.

II. The electric heater *4* placed between the sample elements *1* and *2*. Dimensions $H_y = 100$ mm and $H_z = 100$ mm of all three elements *1*, *2*, *3* (of the studied sample) along the *y* and *z* axes are chosen according to the size of the electric heater *4* made of a permalloy sheet fixed in electrical insulating (dielectric) holders *6*. To obtain the desired electrical resistance R_{heater}=1,57 Ohm of the heater *4*, the permalloy sheet is provided with slots *7*, and the outer elements of the slotted heater are provided with electrical contacts *8* to which wires intended for supplying electric voltage power are connected.

III. Primary temperature measuring transducer made in the form of a thermocouple *5* butt welded (from Chrome and Copper wires) and placed between the elements *1* and *3* of the studied sample.

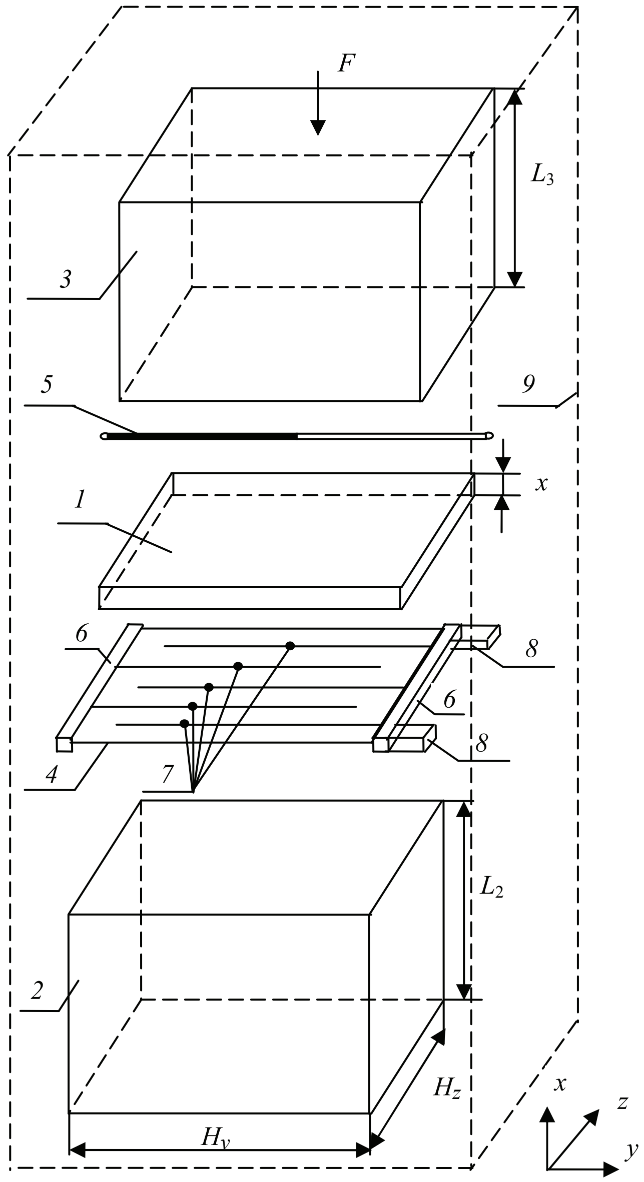

Fig. 1. A scheme illustrating the structure of a measuring cell and the mutual arrangement of its constituent parts

IV. Easily removable thermal insulation shown in Fig. 1 in the form of dashed lines *9*. This easily removable insulation is made of foam plastic in the form of three constituent parts, the internal dimensions of which are 2–3 mm larger than the outer overall dimensions (along the *y, z* axes) of the elements *1*, *2* and *3* of the studied sample.

V. To reduce the thermal resistances that occur at the points of contact:

1) of the sample elements *1* and *2* with the heater *4*;

2) of the sample elements *1* and *3* with each other and with the thermocouple *5*, the structure of the measuring cell involves the use of the constant mass creating the force *F*, shown in Fig. 1 with an arrow, and ensuring mutual pressing of the elements *1*, *2*, *3* to each other, to the heater *4* and the thermocouple *5* with a constant force, which allows to stabilize the value of thermal resistances and minimize the effect of their changes on the results of measuring thermophysical properties, i.e. thermal diffusivity *a*, volume heat capacity $c\rho$ and heat conductivity $\lambda = a \cdot c\rho$.

Constructions of similar measuring devices and their connection schemes to information-measuring and control systems are also considered in [1 – 5].

The mathematical model of the temperature field $T(x, \tau)$ in a flat sample (in the case of using a pulsed plane heat source) on the basis of the classical theory of heat conductivity [1 – 4, 11, 12] can be written as:

$$\frac{\partial T(x,\tau)}{\partial \tau} = a\frac{\partial^2 T(x,\tau)}{\partial x^2}, \quad \tau > 0, \quad 0 < x < \infty; \tag{1}$$

$$T(x,0) = T_0 = 0; \tag{2}$$

$$-\lambda\frac{\partial T(0,\tau)}{\partial x} = q(\tau,\tau_{\text{pulse}}) = q_c\left[h(\tau) - h(\tau-\tau_{\text{pulse}})\right]; \tag{3}$$

$$T(\infty,\tau) = T_0 = 0, \tag{4}$$

where $a = \dfrac{\lambda}{c\rho}$ is the thermal diffusivity coefficient; $q(\tau,\tau_{\text{pulse}})$ is pulse plane heat source with duration τ_{pulse}; q_c is the heat flux supplied to the sample through the surface $x = 0$ during the time interval $0 < \tau \le \tau_{\text{pulse}}$ $h(\tau)$, $h(\tau-\tau_{\text{pulse}})$ are the individual asymmetric step functions given by the ratio [12]:

$$h(\tau) = \begin{cases} 0 & \text{at } \tau < 0, \\ 1 & \text{at } \tau \ge 0; \end{cases}$$

$$h(\tau-\tau_{\text{pulse}}) = \begin{cases} 0 & \text{at } \tau < \tau_{\text{pulse}}, \\ 1 & \text{at } \tau \ge \tau_{\text{pulse}}; \end{cases} \tag{5}$$

τ_{pulse} is the heat pulse duration of $q(\tau,\tau_{\text{pulse}})$.

The ratio used in the mathematical model is graphically illustrated in Fig. 2.

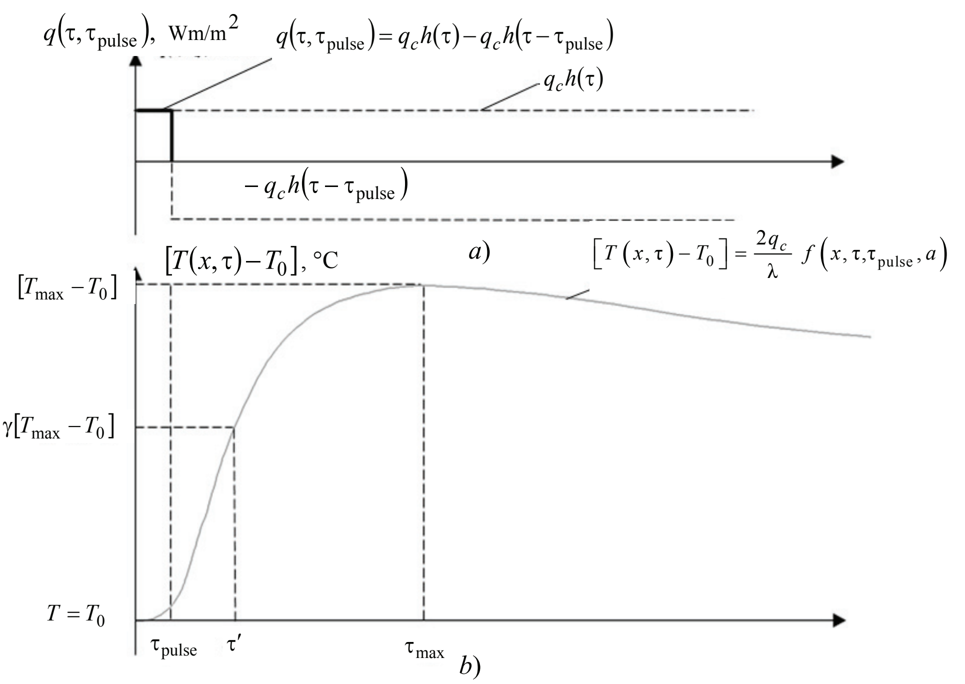

Fig. 2. The change of physical quantities in time τ:

a – the heat pulse $q(\tau,\tau_{\text{pulse}}) = q_c h(\tau) - q_c h(\tau-\tau_{\text{pulse}})$ representing the algebraic sum of step functions $q_c h(\tau) u - q_c h(\tau-\tau_{\text{pulse}})$;

b – the temperature difference $[T(x,\tau) - T_0]$ at a distance *x* from the pulse plane heat source

The solution of the boundary value problem (1) – (4) with the simpler boundary condition (3) in the form

$$-\lambda \frac{\partial T(0,\tau)}{\partial x} = q_c h(\tau),$$

according to the monograph [4] can be written as follows

$$T(x,\tau) - T_0 = \frac{2q_c}{\lambda}\sqrt{a\tau}\ \mathrm{ierfc}\frac{x}{2\sqrt{a\tau}}. \qquad (6)$$

Using the superposition principle [11, 12] and the known ratio (6), the solution of the boundary value problem (1) – (4) stated above takes the following form:

$$T(x,\tau) - T_0 = \frac{2q_c}{\lambda} f\left(x,\tau,\tau_{\text{pulse}},a\right), \qquad (6a)$$

where

$$f\left(x,\tau,\tau_{\text{pulse}},a\right) = \begin{cases} \sqrt{a\tau}\ \mathrm{ierfc}\left(\frac{x}{2\sqrt{a\tau}}\right) & \text{at } 0<\tau\le\tau_{\text{pulse}}; \\ \sqrt{a\tau}\ \mathrm{ierfc}\left(\frac{x}{2\sqrt{a\tau}}\right) - \sqrt{a\left(\tau-\tau_{\text{pulse}}\right)}\times \\ \times\mathrm{ierfc}\left(\frac{x}{2\sqrt{a\left(\tau-\tau_{\text{pulse}}\right)}}\right) & \text{at } \tau>\tau_{\text{pulse}}, \end{cases} \qquad (7)$$

$\mathrm{ierfc}(u) = \int_u^\infty \mathrm{erfc}(W)\,dW = \frac{1}{\sqrt{\pi}}e^{-u^2} - u\,\mathrm{erfc}(u)$ is a special function [11] which is an integral of the function $\mathrm{erfc}(W) = 1-\mathrm{erf}(W)$; $\mathrm{erf}(W) = \frac{2}{\sqrt{\pi}}\int_0^W e^{-W^2}dW$ is a Gauss error function [11, 12].

For the instants of time $\tau \ge \tau_{\text{pulse}}$ the solution (6a) with consideration of (7), takes the following form

$$T(x,\tau) - T_0 = = \frac{q_c x}{\lambda}\left[\frac{\mathrm{ierfc}\left[U(\tau)\right]}{U(\tau)} - \frac{\mathrm{ierfc}\left[U\left(\tau-\tau_{\text{pulse}}\right)\right]}{U\left(\tau-\tau_{\text{pulse}}\right)}\right], \qquad (8)$$

where $U(\tau) = \frac{x}{2\sqrt{a\tau}}$; $U\left(\tau-\tau_{\text{pulse}}\right) = \frac{x}{2\sqrt{a\left(\tau-\tau_{\text{pulse}}\right)}}$

is dimensionless functions depending on $x, \tau, \tau_{\text{pulse}}, a$, and

$$U\left(\tau-\tau_{\text{pulse}}\right) = \frac{x}{2\sqrt{a\left(\tau-\tau_{\text{pulse}}\right)}} = \frac{x}{2\sqrt{a\tau\left(\frac{\tau-\tau_{\text{pulse}}}{\tau}\right)}} = $$
$$= U(\tau)\sqrt{\frac{\tau}{\tau-\tau_{\text{pulse}}}}.$$

The dependency graph (8) is shown in Fig. 2*b*. Fig. 2*b* shows that the change in the temperature difference $\left[T(x,\tau)-T_0\right]$ calculated by formula (8) at the instant of time $\tau = \tau_{\max}$ reaches the maximum value $\left[T_{\max}-T_0\right] = \left[T(x,\tau_{\max})-T_0\right]$, and this moment of time $\tau=\tau_{\max}$ corresponds to the definite value of the dimensionless function

$$U^m = U\left(\tau_{\max}\right) = \frac{x}{2\sqrt{a\tau_{\max}}}.$$

The traditional approach to carrying out the experiment and subsequent processing of the obtained data in the measurement of thermophysical properties by **the plane "instantaneous" heat source method** consists of the following stages [1 – 4]:

1) a sample of the studied material is made in the form of three plates, a flat heater and a thermocouple are placed between these plates (see Fig. 1), and then the uniform distribution of the temperature field $T(x,\tau) = T_0 = \text{const}$ inside the sample of the studied material is expected to be achieved;

2) for a determined period of time $0<\tau\le\tau_{\text{pulse}}$ a constant electric power *P* is applied to a flat electric heater with area *S* and a change in the temperature difference is recorded from the thermocouple signal $\left[T(x,\tau)-T_0\right]$ in the time τ;

3) the maximum value of the temperature difference $\left[T_{\max}-T_0\right] = \left[T(x,\tau_{\max})-T_0\right]$ and the value of the time moment $\tau=\tau_{\max}$ corresponding to this maximum value $\left[T_{\max}-T_0\right]$ are determined according to the obtained experimental data;

4) according to the obtained values of $\tau_{\max}$, $T_{\max}-T_0$, taking into account the known values of the distance *x* and the total amount of heat Q_n supplied to the sample, the required values of the thermal diffusivity coefficient α and heat conductivity λ of the studied material are calculated from the known formulas

$$a = \frac{x^2}{2\tau_{\max}}, \qquad \lambda = \frac{Q_n}{T_{\max} x\sqrt{2\pi e}}.$$

The traditional procedure for carrying out the experiment and processing the obtained data has the following typical drawbacks:

1) high relative error in determining the time moment $\tau = \tau_{max}$ (about 15 – 20 %),

2) lack of recommendations for the choice of:

– the optimal conditions for processing experimental data;

– the optimal thickness x of the middle sample plate;

– the optimal value of the heat pulse duration τ_{pulse}.

The developed technique for carrying out the experiment and processing experimental data

Using the technique for carrying out the experiment and then processing the obtained data (when measuring thermophysical properties by **the pulse plane heat source method**) developed and described in this paper, a sample of the studied material is made in the form of three plates between which a flat heater and a thermocouple are placed. After the uniform distribution of the temperature field $T(x,\tau) = T_0 = \text{const}$ is achieved in the sample, a constant power pulse P is applied to the flat electric heater with an area S for a determined period of time $0 < \tau \le \tau_{pulse}$, and an array of changes in the time of the temperature difference $[T(x,\tau) - T_0]$ is recorded in the thermocouple. According to the obtained array, the maximum temperature difference $[T_{max} - T_0]$ in the sample and the corresponding value of the time moment τ_{max} are determined, using which (taking into account the known x and $q_c = P/(2S)$) the thermal diffusivity coefficient α and the heat conductivity λ are calculated for the studied material according to the formulas (10) and (11) below from the experimental data corresponding to the optimal values of the dimensionless parameter γ.

The methodology proposed by the authors of this article introduce a dimensionless parameter

$$\gamma = \left(T(x,\tau) - T_0\right) \Big/ \left(T_{max} - T_0\right), \qquad (9)$$

representing the ratio of the current temperature difference value $[T(x,\tau) - T_0]$ to the maximum value of this difference $[T_{max} - T_0] = [T(x,\tau_{max}) - T_0]$ at the time moment $\tau = \tau_{max}$.

Fig. 2*b* shows that each temperature difference value $\gamma\left(T_{max} - T_0\right) = T\left(x,\tau'\right) - T_0$, and consequently, each value γ, in the interval $0 < \tau < \tau_{max}$ corresponds to a particular time moment τ' and the value of the dimensionless function $U\left(\tau'\right) = x \Big/ \left(2\sqrt{a\tau'}\right)$.

In order to determine the value of the dimensionless function $U\left(\tau'\right) = x \Big/ \left(2\sqrt{a\tau'}\right)$ corresponding to the given value of the dimensionless parameter γ from the data obtained during the experimental measurement of thermophysical properties (in the form of temperature differences array $[T(x,\tau) - T_0]$ corresponding to the known time moments τ), the following approach is used. If we substitute $\tau = \tau_{max}$ into the formula (8), we obtain

$$T_{max} - T_0 = T\left(x,\tau_{max}\right) - T_0 =$$

$$= \frac{q_c x}{\lambda}\left[\frac{\text{ierfc}\left[U\left(\tau_{max}\right)\right]}{U\left(\tau_{max}\right)} - \frac{\text{ierfc}\left[U\left(\tau_{max} - \tau_{pulse}\right)\right]}{U\left(\tau_{max} - \tau_{pulse}\right)}\right], \qquad (8a)$$

and after dividing (8) by (8a) the ratio (9) takes the following form

$$\gamma = \frac{T(x,\tau) - T_0}{T_{max} - T_0} =$$

$$= \frac{\dfrac{\text{ierfc}\left[U\left(\tau\right)\right]}{U\left(\tau\right)} - \dfrac{\text{ierfc}\left[U\left(\tau - \tau_{pulse}\right)\right]}{U\left(\tau\right)}}{\dfrac{\text{ierfc}\left[U\left(\tau_{max}\right)\right]}{U\left(\tau_{max}\right)} - \dfrac{\text{ierfc}\left[U\left(\tau_{max} - \tau_{pulse}\right)\right]}{U\left(\tau_{max} - \tau_{pulse}\right)}}. \qquad (9a)$$

During the experiment in the numerical modeling of the measurement process, an array of temperature differences $[T(x,\tau) - T_0]$ was determined, and then, using the search method, the maximum value of this difference $T_{max} - T_0$ was found, the analytical expression for which is obtained from (8) at $\tau = \tau_{max}$ in the form (8a). With the known $T_{max} - T_0$ and given γ the root of the equation (9a) was found as the value of the dimensionless function $U\left(\tau'\right)$. Considering the known value of the dimensionless function $U\left(\tau'\right)$ and the known corresponding values of x and τ', on the basis of the ratio $U\left(\tau'\right) = x \Big/ \left(2\sqrt{a\tau'}\right)$, the sought value of the thermal diffusivity coefficient has been calculated by the formula

$$a = x^2 \Big/ \left(4\tau' \left(U\left(\tau'\right)\right)^2\right). \qquad (10)$$

Taking into account (8) and (9), the formula for calculating the heat conductivity takes the following form

$$\lambda = \Phi\left[U\left(\tau'\right), \tau_{\text{pulse}}, \tau'\right] q_c x \Big/ \left[T\left(x, \tau'\right) - T_0\right], \quad (11)$$

where

$$\Phi\left[U\left(\tau'\right), \tau_{\text{pulse}}, \tau'\right] = \text{ierfc}\left[U\left(\tau'\right)\right] \Big/ U\left(\tau'\right) -$$

$$-\text{ierfc}\left[U\left(\tau'\right)\sqrt{\tau' \Big/ \left(\tau' - \tau_{\text{pulse}}\right)}\right] \Big/ \left(U\left(\tau'\right)\sqrt{\tau' \Big/ \left(\tau' - \tau_{\text{pulse}}\right)}\right).$$

The choice of the optimal values of the dimensionless parameter γ and the heat pulse duration τ_{pulse}

In accordance with the theory of errors [1, 14, 15] and the available experience of solving similar problems [5 – 10, 13, 16 – 22], after making the logarithm of the dependence (10) and the subsequent definition of the differential from the left and right parts we get the following:

$$\ln a = 2\ln x - \ln 4 - \ln \tau' - 2\ln U\left(\tau'\right);$$

$$d\ \ln a = 2d\ \ln x - d\ \ln 4 - d\ \ln \tau' - 2d\ \ln U\left(\tau'\right);$$

$$\frac{da}{a} = 2\frac{dx}{x} - \frac{d4}{4} - \frac{d\tau'}{\tau'} - 2\frac{dU\left(\tau'\right)}{U\left(\tau'\right)}. \qquad (12)$$

According to the theory of errors, it has been carried out [1, 14, 15]:

1) the replacement of differentials $da \approx \Delta a$, $dx \approx \Delta x$, $d\tau' \approx \Delta\tau'$, $dU\left(\tau'\right) \approx \Delta U\left(\tau'\right)$ by absolute errors Δa, Δx, $\Delta\tau'$, $\Delta U\left(\tau'\right)$;

2) taking into account that the differential of the constant is $d4 = 0$;

3) the replacement of the signs "–"by signs "+" in (12), and the formula for calculating the so-called marginal estimation of the relative error in measuring the thermal diffusivity has been obtained

$$\left(\frac{\Delta a}{a}\right)_{\text{marg}} = 2\frac{\Delta x}{x} + \frac{\Delta\tau'}{\tau'} + 2\frac{\Delta U\left(\tau'\right)}{U\left(\tau'\right)}$$

or

$$\left(\delta a\right)_{\text{marg}} = 2\delta x + \delta\tau' + 2\delta U\left(\tau'\right),$$

where

$$\delta a_{\text{marg}} = \frac{\Delta a}{a};\ \delta x = \frac{\Delta x}{x};\ \delta\tau' = \frac{\Delta\tau'}{\tau'};\ \delta U\left(\tau'\right) = \frac{\Delta U\left(\tau'\right)}{U\left(\tau'\right)} -$$

expresses relative errors in the determination of the corresponding physical quantities a, x, τ', $U\left(\tau'\right)$.

After the transition (according to recommendations [1, 14, 15]) from the marginal estimation $(\delta a)_{\text{marg}}$ to the root-mean-square (**RMS**) estimation $(\delta a)_{\text{RMS}}$ of the error in determining the thermal diffusivity coefficient, we obtain

$$(\delta a)_{\text{RMS}} = \sqrt{4(\delta x)^2 + (\delta\tau')^2 + 4\left[\delta U\left(\tau'\right)\right]^2}. \qquad (13)$$

Let us consider the procedure for determining the errors included in the last expression (13) in a more detailed way. Bearing in mind that the value of the time moment depends on the dimensionless parameter γ, i.e. $\tau' = \tau'\left(\gamma\right)$,, we get

$$\delta\left[U\left(\tau'\left(\gamma\right)\right)\right] = \delta U'\left(\gamma\right) \approx$$

$$\approx \frac{dU'\left(\gamma\right)}{U'\left(\gamma\right)} = \frac{1}{U'}\frac{dU'}{d\gamma}d\gamma \approx \frac{1}{U'}\frac{dU'}{d\gamma}\Delta\gamma\ .$$

To determine the absolute error $\Delta\gamma$ we can transform the formula (9) (by analogy with the above)

$$(\delta\gamma)_{\text{RMS}} = \sqrt{\left[\frac{\Delta T}{T\left(x,\tau'\right) - T_0}\right]^2 + \left[\frac{\Delta T}{T_{\max} - T_0}\right]^2} =$$

$$= \delta\left(T_{\max} - T_0\right)\sqrt{\frac{1}{\gamma^2} + 1}$$

or

$$\Delta\gamma = \gamma\ \delta\left(T_{\max} - T_0\right)\sqrt{\frac{1}{\gamma^2} + 1} = \delta\left(T_{\max} - T_0\right)\sqrt{1 + \gamma^2}, \qquad (14)$$

where ΔT is an absolute error in measuring temperature difference; $\delta\left(T_{\max} - T_0\right) = \Delta T \Big/ \left(T_{\max} - T_0\right)$ is a relative error in measuring the maximum value of the temperature difference $\left(T_{\max} - T_0\right)$; $\Delta\gamma, (\delta\gamma)_{\text{RMS}}$ is absolute and root-mean-square relative errors in determining the dimensionless parameter γ from the experimentally measured values of the temperature differences $\left[T\left(x,\tau'\right) - T_0\right]$ and $\left[T_{\max} - T_0\right]$.

The relative error $\delta\tau'$ in the determination of the time moment τ' in (13) is also related to the errors in measuring the temperature differences $\left[T\left(x,\tau'\right)-T_0\right]$.

From the ratio $\left.\dfrac{\partial\left[T\left(x,\tau'\right)-T_0\right]}{\partial\tau}\right|_{\tau=\tau'} \approx \dfrac{\Delta T}{\Delta\tau}$ we get

$$\delta\tau' = \frac{\Delta\tau'}{\tau'} = \frac{\Delta T}{\tau'\left.\left\{\dfrac{\partial\left[T\left(x,\tau'\right)-T_0\right]}{\partial\tau'}\right\}\right|_{\tau=\tau'}}, \tag{15}$$

where $\Delta\tau'$, $\delta\tau'$ is absolute and relative errors in determining the time moment corresponding to a given value of the dimensionless parameter γ.

Substituting (14), (15) into the formula (13), we obtain the following ratio

$$(\delta a)_{\text{RMS}} = \sqrt{4(\delta x)^2 + \left[\frac{\Delta T}{\tau'\left\{\dfrac{d\left[T\left(x,\tau\right)-T_0\right]}{d\tau'}\right\}}\right]^2 + \left[\frac{1}{U'}\,\frac{dU'}{d\gamma}\,\sqrt{\gamma^2+1}\;\delta\left(T_{\max}-T_0\right)\right]^2}, \tag{16}$$

which is used in further calculations to get (for the case of measuring the thermal diffusivity coefficient) the optimal values:

1) the dimensionless parameter γ;

2) the basic construction size x of the middle plate of the studied sample (see Fig. 1).

Then the work was done to obtain a ratio for calculating the root-mean-square estimation of the relative error $(\delta\lambda)_{\text{RMS}}$ in measuring the heat conductivity λ. Taking into account that for each known heat pulse duration the values:

1) the time moment $\tau'=\tau'(\gamma)$;

2) the dimensionless function $U\left(\tau'(\gamma)\right)=U'(\gamma)$

depend on the dimensionless parameter γ, the previously obtained formula (11) for calculating the heat conductivity has been transformed to the form

$$\lambda = \frac{q_c x}{\left[T\left(x,\tau(\gamma)\right)-T_0\right]}\,F(\gamma), \tag{17}$$

where $F(\gamma)\equiv\Phi\left[U\left(\tau'(\gamma)\right),\,U\left(\tau'(\gamma)-\tau_{\text{pulse}}\right)\right]$.

Then, according to the recommendations of the theory of errors [1, 14, 15], after making the logarithm of the dependence (17) and the subsequent determination of the differentials from the left and right parts, we obtain:

$$\ln\lambda = \ln q_c + \ln x + \ln F[\gamma] - \ln[T(x,\tau)-T_0];$$

$$d\ln\lambda = d\ln q_c + d\ln x + d\ln F[\gamma] - d\ln[T(x,\tau)-T_0];$$

$$\frac{d\lambda}{\lambda} = \frac{dq_c}{q_c} + \frac{dx}{x} + \frac{dF[\gamma]}{F[\gamma]} - \frac{d[T(x,\tau)-T_0]}{[T(x,\tau)-T_0]}.$$

Having replaced the differentials $d\lambda\approx\Delta\lambda$, $dx\approx\Delta x$, $dq_c=\Delta q_c$, $dF[\gamma]\approx\Delta F[\gamma]$, $d\left[T\left(x,\tau'\right)-T_0\right]\approx\Delta\left[T\left(x,\tau'\right)-T_0\right]$ $d\approx\Delta$ with the corresponding absolute errors $\Delta\lambda$, Δx, $\Delta\gamma$, $\Delta F[\gamma]$, Δq_c, $\Delta\left[T\left(x,\tau'\right)-T_0\right]$ as adopted in the theory of errors [1, 14, 15], we obtain

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta q_c}{q_c} + \frac{\Delta x}{x} + \frac{\Delta F[\gamma]}{F[\gamma]} - \frac{\Delta[T(x,\tau)-T_0]}{[T(x,\tau)-T_0]}.$$

Having changed signs "–" with signs "+", we obtain an expression for calculating the so-called marginal estimation [1, 14, 15] of the relative error in measuring heat conductivity

$$\left(\frac{\Delta\lambda}{\lambda}\right)_{\text{marg}} = \frac{\Delta q_c}{q_c} + \frac{\Delta x}{x} + \frac{\Delta F[\gamma]}{F[\gamma]} + \frac{\Delta[T(x,\tau)-T_0]}{[T(x,\tau)-T_0]}$$

or

$$(\delta\lambda)_{\text{marg}} = \delta q_c + \delta x + \delta F[\gamma] + \delta\left[T(x,\tau)-T_0\right], \tag{18}$$

where

$$\delta\lambda_{\text{marg}} \approx \frac{\Delta\lambda}{\lambda},\quad \delta x\approx\frac{\Delta x}{x},\quad \delta q_c = \frac{\Delta q_c}{q_c},\ \delta F[\gamma],$$

$\delta\left[T\left(x,\tau'\right)-T_0\right]$ are relative errors in determining the corresponding values λ, x, q_c, $F(\gamma)$, $\left[T\left(x,\tau'\right)-T_0\right]$.

By analogy with the above procedure for determining errors [1, 14, 15], we obtain:

$$\delta F[\gamma]=\frac{1}{F[\gamma]}\frac{\partial F[\gamma]}{\partial\gamma}\Delta\gamma;$$

$$\Delta\gamma=\gamma\delta\left(T_{\max}-T_0\right)\sqrt{\frac{1}{\gamma^2}+1}=\delta\left(T_{\max}-T_0\right)\sqrt{1+\gamma^2},$$

$$\delta\left[T(x,\tau)-T_0\right]=\frac{\Delta T}{T(x,\tau)-T_0},$$

where

$$\Delta T\equiv\Delta\left[T\left(x,\tau'\right)-T_0\right]$$

is an absolute error in measuring temperature differences $\left[T\left(x,\tau'\right)-T_0\right]$.

Taking into account the above calculations, the expression (18) takes the following form

$$\left(\delta\lambda\right)_{\text{marg}}=\delta q_c+\delta x+\frac{1}{F(\gamma)}\frac{\partial F(\gamma)}{\partial\gamma}\times$$

$$\times\sqrt{1+\gamma^2}\,\delta\left(T_{\max}-T_0\right)+\frac{\Delta T}{T(x,\tau)-T_0}.$$

After the transition (according to recommendations [1, 5, 6, 14, 15]) from the marginal estimation $(\delta\lambda)_{\text{marg}}$ to the root-mean-square estimation $(\delta\lambda)_{\text{RMS}}$ of the relative error in the determination of heat conductivity, we obtain

$$(\delta\lambda)_{\text{RMS}}=\sqrt{(\delta q_c)^2+(\delta x)^2+\left\{\frac{1}{F(\gamma)}\frac{\partial F(\gamma)}{\partial\gamma}\sqrt{1+\gamma^2}\,\delta\left(T_{\max}-T_0\right)\right\}^2+\left(\frac{\Delta T}{T(x,\tau)-T_0}\right)^2}. \tag{19}$$

To reduce the errors in the measurement of heat conductivity $(\delta\lambda)_{\text{RMS}}$ during the measuring process, it is desirable to provide such modes of conducting experiments in which the maximum temperature difference in the sample would not change very much, i.e. $\left[T_{\max}-T_0\right]\approx\text{const}$. For that purpose, in each experiment a constant amount of heat $Q_{\text{const}}=2q_c\tau_{\text{pulse}}=\text{const}$ should be brought about to the unit of the sample surface (1 m^2). Here we note that the value of Q_{const} has dimensions J/m^2. Then the power consumed by the heater, ensuring the implementation of the formulated condition $Q_{\text{const}}\approx$ const, can be calculated using the formula $P=Q_{\text{const}}S/\tau_{\text{pulse}}$. In this case, the heat flux (from the heater with power P and area S) supplied to the lower side of the middle plate (indicated by the position *1* in Fig. 1) for a period of time $0\le\tau\le\tau_{\text{pulse}}$ can be calculated by the formula

$$q_c=\frac{P}{2S}. \tag{20}$$

Numerical calculations and experimental studies have shown the following. When using samples of heat insulation materials (with the thickness of the middle plate 2.5 – 5.0 mm, the thermal diffusivity coefficient $a=(1.0-1.5)\cdot10^{-7}$ m^2/s and heat conductivity $\lambda=0.05-0.2$ W/(m·K)) in order to obtain the temperature difference in the range of 3 – 7 °C the amount of heat supplied to a unit surface should be in the range of values $Q_{\text{const}}\approx 50-80$ kJ/m^2.

With the values of $Q_{\text{const}}=55$ kJ/m^2 and the heater area $S=0.01$ m^2, the power consumed by the heater can be calculated by the formula

$$P=\frac{550}{\tau_{\text{pulse}}}. \tag{21}$$

Let us consider the calculation of the error component included in the formula (19).

After making the logarithm of the expression (20), defining the differentials of its left and right parts, and implementing the recommendations of the theory of errors [1, 14, 15], we get:

$$\delta q_c=\sqrt{(\delta P)^2+(\delta S)^2}=\sqrt{\left(\Delta P/P(\tau_{\text{pulse}})\right)^2+\left(\Delta S/S\right)^2}. \tag{22}$$

When performing calculations, the values $P\left(\tau_{\text{pulse}}\right)$ have been calculated by the formula (21). After substitution (22) in (19) we get:

$$(\delta\lambda)_{\text{RMS}}=\sqrt{\left[\frac{\Delta P}{P(\tau_{\text{pulse}})}\right]^2+(\delta S)^2+(\delta x)^2+\left(\gamma^2+1\right)\left(\frac{dF\left(\gamma\right)}{F\left(\gamma\right)d\gamma}\delta\left(T_{\max}-T_0\right)\right)^2+\left(\Delta T/\left(T\left(x,\tau\right)-T_0\right)\right)^2}. \tag{23}$$

Table 1

The dependence of errors $(\delta a)_{\text{RMS}}$, $(\delta\lambda)_{\text{RMS}}$, $\delta_{\text{cp}} = \dfrac{(\delta a)_{\text{RMS}} + (\delta\lambda)_{\text{RMS}}}{2}$ from the value of the heat pulse duration τ_{pulse} with the thickness of the middle plate in the sample *x* = 3,55 mm

Root-mean-square error	τ_{pulse}, s							
	5	10	15	20	25	30	35	40
$(\delta\lambda)_{\text{RMS}}$	5.1105	5.1272	5.1448	5.1933	5.2423	5.3014	5.3704	5.4489
$(\delta a)_{\text{RMS}}$	6.6725	6.4696	6.3493	6.2802	6.2418	6.2223	6.2149	6.2147
δ_{mean}	5.8915	5.7984	5.7521	5.7367	5.742	5.7619	5.7925	5.8318

When calculating according to (23), it was assumed that $\Delta P = 0.5$ W and the relative error in measuring the area *S* of the heater is $\delta S = 0,5\ \%$.

The results of calculating the dependences of errors $(\delta a)_{\text{RMS}}$, $(\delta\lambda)_{\text{RMS}}$ and their arithmetic mean values $\delta_{\text{mean}} = \dfrac{(\delta a)_{\text{RMS}} + (\delta\lambda)_{\text{RMS}}}{2}$ from the heat pulse duration τ_{pulse} are presented in Table 1.

Table 1 shows that with increasing τ_{pulse} the error $(\delta\lambda)_{\text{RMS}}$ monotonously increases, the error $(\delta a)_{\text{RMS}}$ decreases, and their arithmetic mean value δ_{mean} takes the minimum values in the interval 15 < τ_{pulse} < 25 s.

Results of numerical simulation of the root-mean-square relative errors in the measurement of thermal diffusivity coefficient *a* and heat conductivity λ

Using the obtained formulas (16) and (23), the dependences of the root-mean-square relative errors $(\delta a)_{\text{RMS}}$, $(\delta\lambda)_{\text{RMS}}$ at the heat pulse duration $\tau_{\text{pulse}} = 21$ s have been calculated and shown in Fig. 3. The following initial data have been used: $\delta S = 0.5$ %, $Q_{\text{const}} = 55$ kJ/m^2, 5 < τ_{pulse} < 40 s, $a = 1.06 \cdot 10^{-7}$ m^2/s, $\lambda = 0.194$ W/(mK), $\Delta P = 0.5$ W, $x = 2$–8 mm, $\Delta x = 0.1$ mm, $\Delta T = 0.05$ K.

During the research it has become obvious that the minimum values of the relative errors $(\delta a)_{\text{RMS}}$, $(\delta\lambda)_{\text{RMS}}$ depend not only on the dimensionless parameter γ, but also on the distance *x* from the heater placement plane to the thermocouple installation plane, which measures the temperature difference $\left[T\left(x, \tau'\right) - T_0\right]$. In this regard, it has been decided to construct lines of equal error levels on the plane with coordinates γ and *x*. The results of this work are presented in Fig. 4.

The results of calculations presented in Fig. 4 show that (using the initial data in the calculations) acceptable values of the root-mean-square relative errors $(\delta a)_{\text{RMS}}$ of measuring the thermal diffusivity coefficient *a* occur at the values of a dimensionless parameter in the range $0.35 < \gamma^a \le 0.59$ and at the values of the basic construction size of the measuring device within the limits $4.0 < x \le 5.0$ mm. The minimum values of the error $(\delta a)_{\text{RMS}}$ are achieved at $\gamma^a_{\text{opt}} \approx 0.465$ and $x^a_{\text{opt}} \approx 4.5$ mm.

At the same time, acceptable values of the root-mean-square relative errors $(\delta\lambda)_{\text{RMS}}$ in measuring the heat conductivity λ occur in the range of values $0.90 < \gamma \le 1.0$ and $2.4 \le x \le 3.6$. At the same time, the minimum values of the errors $(\delta\lambda)_{\text{RMS}}$ are achieved at $\gamma^\lambda_{\text{opt}} = 1$ and $x^\lambda_{\text{opt}} \approx 3.0$ mm.

Thus, in order to achieve the minimum values of error $(\delta a)_{\text{RMS}}$ and $(\delta\lambda)_{\text{RMS}}$ when measuring the thermal diffusivity coefficient *a* and heat conductivity λ of the studied material, a measuring transducer should be used with the distance between the temperature meter and the heater in the range $3.0 < x < 4.5$ mm, in which connection the following formula can be accepted

$$x_{\text{opt}} = \frac{x^a_{\text{opt}} + x^\lambda_{\text{opt}}}{2} = 3.75 \text{ mm.}$$

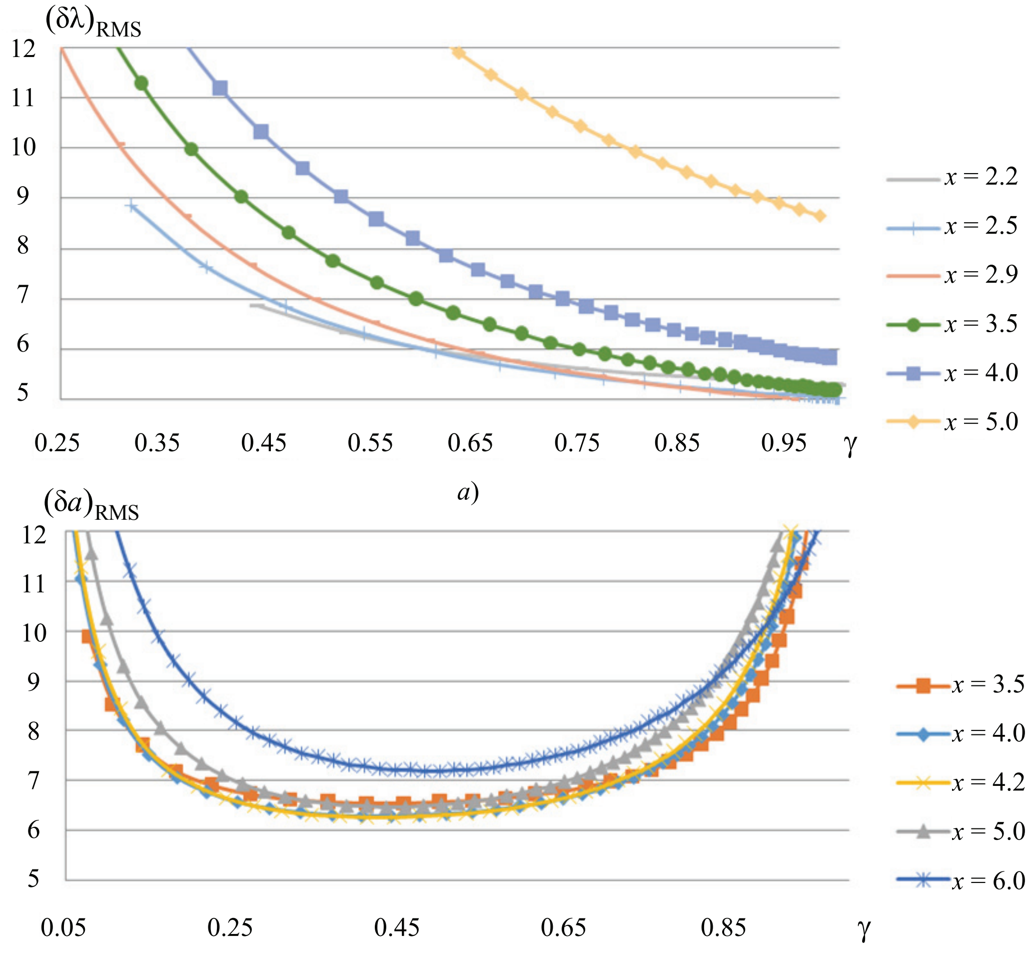

Fig. 3. Dependencies of the root-mean-square relative errors $(\delta\lambda)_{RMS}$ and $(\delta a)_{RMS}$ (on the dimensionless parameter γ for different values of the distance *x* from the location of the plane pulse heat source to the plane in which the thermocouple measuring the temperature difference $\left[T\left(x,\tau'\right)-T_0\right]$ is located) during measurements of:

a – heat conductivity λ; *b* – thermal diffusivity coefficient *a*

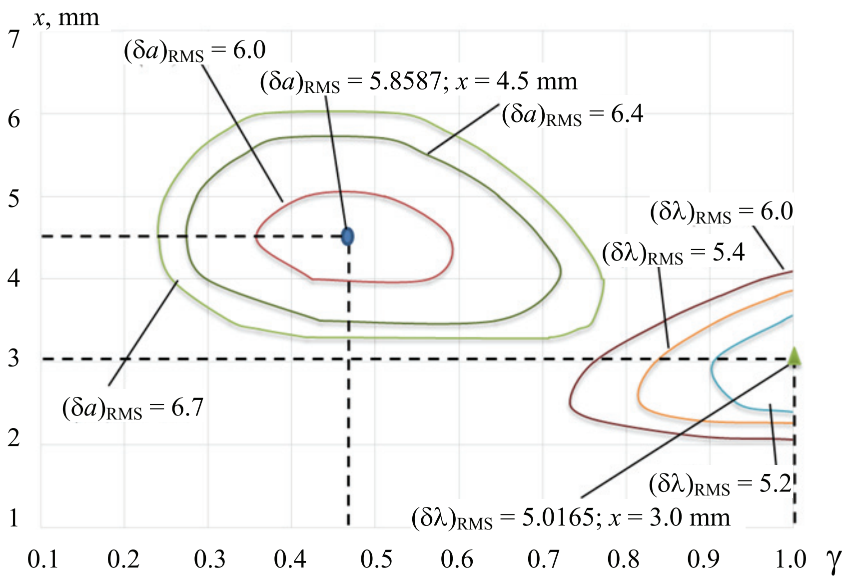

Fig. 4. Lines of equal levels in root-mean-square relative errors $(\delta a)_{RMS}$ and $(\delta\lambda)_{RMS}$ constructed with an optimal heat pulse duration τ_{pulse} of 21 s

To clarify the optimal value of the heat pulse duration τ_{pulse} ensuring the achievement of the minimum values of relative errors $(\delta a)_{RMS}$, $(\delta\lambda)_{RMS}$ and the arithmetic mean values of the errors $\delta_{mean} = \frac{[(\delta a)_{RMS} + (\delta\lambda)_{RMS}]}{2}$ in measuring thermophysical properties a and λ, calculations have been performed using formulas (16) and (23) (for the already determined optimal values), the results of which are shown in Fig. 5.

Fig. 5 shows that when changing the heat pulse duration τ_{pulse} the arithmetic mean value of root-mean-square estimations of relative errors takes on minimal values at $\tau_{pulse}^{opt} \approx 21$ s in the range of $18 < \tau_{pulse} < 24$ s.

The above dependence presented in Fig. 5 may leave a wrong impression that when taking into account the influence of the heat pulse duration τ_{pulse}, the measurement errors decrease only by 0.025 – 0.063 %. In fact, the application of the measurement and data processing methods proposed in this paper allows the arithmetic mean value of the root-mean-square estimations of relative errors $\delta_{mean} = \frac{[(\delta a)_{RMS} + (\delta\lambda)_{RMS}]}{2}$ to be reduced by

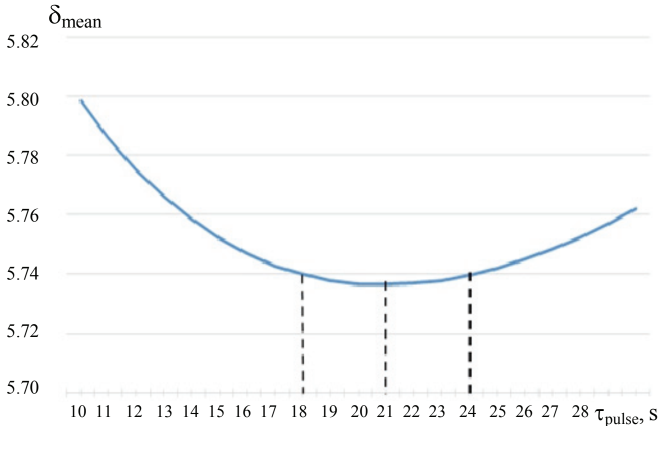

Fig. 5. Dependencies of arithmetic mean values $\delta_{mean} = \frac{[(\delta a)_{RMS} + (\delta\lambda)_{RMS}]}{2}$ of root-mean-square estimations of relative errors $(\delta a)_{RMS}$, $(\delta\lambda)_{RMS}$ for measuring the thermal diffusivity coefficient a and heat conductivity λ on the heat pulse duration τ_{pulse} when processing data using the pulse plane heat source method considered in this article

5 – 8 % compared with the traditional plane "instantaneous" heat source method [1 – 4].

To illustrate this fact we performed calculations of the thermal diffusivity coefficient a and heat conductivity λ values for various heat pulse duration τ_{pulse} values using:

– the calculated ratios (10) and (11) proposed in this article;

– the calculated ratios [1 – 4]

$$a_{inst} = \frac{x^2}{2\tau_{max}}; \quad \lambda_{inst} = \frac{Q_{const} x_0}{2\sqrt{2\pi e}\ \tau_{max} T_{max}}, \qquad (24)$$

used in the implementation of the traditional plane "instantaneous" heat source method. In these calculations the exact values $a_{exact} = 1.06 \cdot 10^{-7}$ m^2/s, $\lambda_{exact} = 0.194$ W/(mK), $x_{opt} = 3.75$ mm, have been used and the power value has been calculated by the formula (21).

After calculating the values of a and λ by the formulas (10) and (11), as well as a_{inst} and λ_{inst} according to the formulas (24), the errors

$$\delta a = \frac{a - a_{exact}}{a_{exact}} \cdot 100\ \%, \quad \delta\lambda = \frac{\lambda - \lambda_{exact}}{\lambda_{exact}} \cdot 100\ \%,$$

$$\delta a_{inst} = \frac{a_{inst} - a_{exact}}{a_{exact}} \cdot 100\ \%,$$

$$\delta\lambda_{inst} = \frac{\lambda_{inst} - \lambda_{exact}}{\lambda_{exact}} \cdot 100\ \%,$$

have been calculated and then the arithmetic mean values $\bar{\delta} = \frac{[\delta a + \delta\lambda]}{2}$ and $\bar{\delta}_{inst} = \frac{[\delta a_{inst} + \delta\lambda_{inst}]}{2}$ have been found.

As a result, the graphs $\bar{\delta} = f_1(\tau_{pulse})$ and $\bar{\delta}_{inst} = f_2(\tau_{pulse})$ presented in Fig. 6 have been built.

The graphs presented in Fig. 6 show that the following results have been obtained when numerically simulating the measurement of thermophysical properties:

1) when using the plane pulse heat source method proposed in the article, the arithmetic mean values of the data processing errors $\bar{\delta} = f_1(\tau_{pulse})$ do not exceed 1 %;

2) when processing data with calculation ratios (24), which are the basis of the traditional plane "instantaneous" heat source method [1 – 4], the arithmetic mean values of data processing errors

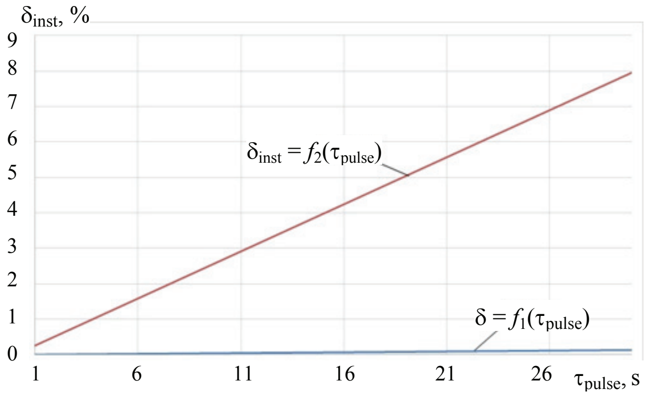

Fig. 6. Comparison of experimental data processing errors $\overline{\delta}_{inst} = f_2(\tau_{pulse})$ in the case of using the traditional plane "instantaneous" heat source method and experimental data processing errors $\overline{\delta} = f_1(\tau_{pulse})$ using the plane pulse heat source method considered in this article

$\overline{\delta}_{inst} = \frac{[\delta a_{inst} + \delta\lambda_{inst}]}{2} = f_2(\tau_{pulse})$ reach 5–8 % with the heat pulse duration in the range $17 < \tau_{pulse} < 30$ s.

Conclusion

Thus:

1) using the approach for choosing the optimal value of the dimensionless parameter γ, the rational construction size x of the middle plate in the sample of the studied material and the heat pulse duration τ_{pulse} proposed in the article reduces the relative measurement errors of thermal diffusivity coefficient and heat conductivity from 10–12 to 4–5 %.

2) to achieve the minimum values of error $(\delta a)_{RMS}$ in measuring the thermal diffusivity coefficient a, a sample of the studied material with a plate thickness $4.0 \le x \le 5.0$ mm should be used and the experimental data should be processed at $0.45 < \gamma \le 0.48$;

3) to ensure the minimum error values $(\delta\lambda)_{RMS}$ when measuring heat conductivity λ, it is required to use a sample with a thickness of the middle plate $2.8 \le x \le 3.2$ mm, and to process the experimental data at $0.95 < \gamma \le 1.0$;

4) if it is necessary to simultaneously measure the thermal diffusivity coefficient a and heat conductivity λ in one experiment, then the thickness x of the middle plate of the sample from the studied material should be chosen from the range $3.5 \le x \le 4.0$ mm, that ensures the relative measurement errors $(\delta a)_{RMS} \approx 5.9$–6.7 % and $(\delta\lambda)_{RMS} \approx 5.0$–6.0 %.

Note that measuring the thermophysical properties of the studied material, the thermal diffusivity coefficient a and heat conductivity λ of which differ from those used in this article (in the source data of the above calculations), should be done as follows:

1) by conducting preliminary measurements it is necessary to experimentally determine the indicative values of the thermal diffusivity coefficient a_{indic} and heat conductivity λ_{indic} of the studied material;

2) by analogy with the method described in the article above it is necessary to carry out calculations (with a_{indic} and λ_{indic} values found) in order to determine (clarify) the optimal values of the parameter γ_{onm} and construction sizes x^a_{opt} and x^λ_{opt} of the middle plate used to measure the thermal diffusivity coefficient a and heat conductivity λ;

3) by making samples with two middle plates with thickness x^a_{opt} and x^λ_{opt}, or with one middle plate with thickness $x_{opt} = \left(x^a_{opt} + x^\lambda_{opt}\right)/2$;

4) by conducting a series of experiments (with fabricated samples) to carry out measurements and subsequent processing of the obtained data, and as a result to obtain the values of the desired thermal diffusivity coefficient a and heat conductivity λ of the studied material.

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