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# Mechanism and kinetics of separation of impurity particles with different densities in a rapid gravity flow of granular material

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**Abstract:** The efficiency of the separation process of impurity particles with different densities in a rapid gravity flow of monodisperse granular material has been studied. The study was carried out by mathematical modeling of particle concentration distribution dynamics on a rough chute using a mathematical model that takes into account the kinetics of convection, mixing, segregation and quasi-diffusion separation effects. During the research process, the relative density of impurity particles in the range of 0.2–4.0 was varied with respect to the base component particle density, and this confirms the defining role of the quasi-diffusive separation effect. It was found that the concentration distribution profiles of low- and high-density particles have a shape similar to the shape of the profiles of the volume fractions of voids and solids, respectively. With the same degree of difference between the light and heavy impurity particles from the base component particles, the light particles have a higher tendency to separate. The tendency to separation with increasing degree of difference between the density of the impurity particles and the density of the base component particles increases more intensively for the impurity particles with low density. It is found that with decreasing concentration of impurity particles in the flow, the intensity of their separation increases for particles with low density and decreases for particles with high density. A binary mixture of homogeneous particles with the minimum volume content of low density particles is characterized by the highest tendency to particle density separation.

**Keywords:** granular material; rapid gravity flow; particle density separation; segregation; quasi-diffusion separation (migration); shear rate; void volume fraction.

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# Механизм и кинетика сепарации примеси частиц различной плотности в быстром гравитационном потоке зернистого материала

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**Аннотация:** Проведено исследование эффективности процесса сепарации примеси частиц различной плотности в быстром гравитационном потоке монодисперсного зернистого материала. Исследование выполнено методом математического моделирования динамики распределения концентрации примеси частиц на шероховатом скате с использованием математической модели, учитывающей кинетику эффектов конвекции, перемешивания, сегрегации и квазидиффузионной сепарации. Исследование, проведенное при варьировании относительной плотностью частиц примеси в интервале 0,2…4,0 по отношению к плотности частиц базового компонента, подтверждает определяющую роль эффекта квазидиффузионной сепарации. Установлено, что профили распределения концентрации примеси частиц с малой и высокой плотностью имеет форму, аналогичную форме профилей объемных долей пустот и твердой фазы, соответственно. При одинаковой степени отличия частиц легких и тяжелых примесей от частиц базового компонента более высокую склонность к сепарации имеют легкие частицы. Склонность к сепарации с увеличением степени отличия плотности частиц примеси от плотности частиц базового компонента повышается более интенсивно для примеси частиц с малой плотностью. Установлено, что с уменьшением концентрации частиц примеси в потоке интенсивность их сепарации повышается для частиц с малой плотностью и снижается для частиц с высокой плотностью. Наиболее высокой склонностью к сепарации частиц по плотности характеризуется бинарная смесь однородных по размеру частиц с минимальным объемным содержанием примеси частиц, имеющих малую плотность.

**Ключевые слова:** зернистый материал; быстрое гравитационное течение; сепарация частиц по плотности; сегрегация; квазидиффузионная сепарация (миграция); скорость сдвига; объемная доля пустот.

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## 1. Introduction

Rapid shear flows of granular materials are one of the most common types of flows occurring in natural environments and technological processes [1–5]. In most cases such flows are formed under the influence of gravity and are referred to as rapid gravity flows [4]. The specific dynamics of rapid gravity flows is a consequence of their characteristic mechanism of generating shear and normal stresses, in which inertial forces resulting from the exchange of particles by impact impulses play a decisive role [6]. Such kinds of flows are formed on gravity descents, repose slopes of materials in bunkers and bunker storages, inclined decks of separators and mixers, in heaps of material in rotating tubes and drums and many other cases.

Under conditions of fast gravity flow the exchange of particles by shock impulses is accompanied by dilatancy of granular medium, the degree of which depends on shear rate and lithostatic pressure value [7–9]. As a result, in a fast gravity flow there is a high inhomogeneity of shear rate and void volume fraction by bed height, which is especially pronounced in a thin-bed flow. Because of the dilatancy of a granular medium in a fast gravity flow favourable conditions for reciprocal movement of particles with manifestation of the effects of their mixing and separation on a set of distinctive properties are created. The effects of separation and mixing in this case are so significant [10–12] that they have a notable impact on the dynamics of the flow of granular media and the kinetics of technological processes. Research shows [7, 13–16] that for the intensity of the separation effect, not only the degree of heterogeneity of the physical and mechanical properties of the particles and the conditions of their interaction in the flow, determined by local values of shear rate and volume fraction of the solid phase, but also the nature of those changes in the height of the moving bed are important. Therefore, in modeling of separation it is reasonable to consider its flux as a set of fluxes. The magnitude of one of the separation fluxes is determined by the degree of local nonuniformity of shear flow in a granular medium (nonuniformity of particle properties with respect to local conditions of their interaction). The intensity of the other separation flow is determined by the degree of spatial heterogeneity of structural and kinematic parameters of the flow.

A large number of works, e.g. [7, 13–20], are devoted to investigation of kinetic regularities of separation and mixing effects in fast gravity flows of granular media consisting of cohesionless inelastic nonuniform particles. However, research results of most of these works are characterized by very limited predictive capabilities. For example, according to one of the recent works [15], to predict particle size and density separation dynamics, knowledge of partial kinetic and contact stress coefficients, as well as segregation drag and quasi-diffusion mixing coefficients as functions of solid phase concentration and shear rate, is necessary. It is extremely problematic to obtain such information in the absence of appropriate scientifically based methods. In addition, in [15] kinetic stress gradient has been used as the driving force of separation due to local and spatial heterogeneity of gravity flow. Such an undifferentiated approach to estimate the effects of local and spatial heterogeneity of shear flow in the separation process makes it difficult to predict its kinetics in conditions of high homogeneity of structural and kinematic flow parameters. At the same time, in this work, as in [18], an autonomous (differentiated) approach to modeling of particle separation by size and density is implemented, despite the fact that complex influence of particle properties on the kinetics of alternative separation effects seems obvious [20]. For example, according to the research carried out in [20], the alternative separation effects in gravity flow are represented as ascent and percolation.. In this case, the effects of both ascent and percolation. of particles are determined as a result of complex consideration of their size and density differences. The purpose of the finite element method (DEM) research is to identify combinations of relative values of particle diameters and densities that minimise the tendency of the granular material to segregate.

The papers [7, 14, 16] devoted to investigation of the dynamics of particle segregation in a fast thin-bed gravity flow with high inhomogeneity of shear rate and void volume fraction in the flow focus attention on the study of the quasi-diffusion separation effect. It is found that the effect of quasi-diffusion separation of particles according to the complex of physical and mechanical properties is shown along with the effects of quasi-diffusive mixing and hydro-mechanical separation, traditionally called segregation. In this case the effects of hydro-mechanical segregation and quasi-diffusion separation are due to, respectively, relaxation of local stress concentration and spatial heterogeneity of interaction conditions of particles in the gravity flow.

The effects of segregation and quasi-diffusion separation have principally different physical nature and are caused by the action of principally different driving forces [7, 14, 16]. The segregation mechanism consists in relaxation of excess stresses generated by nonuniform particles in shear flow [11, 21, 22]. The excess stresses are calculated with respect to the stresses generated by particles of a conventionally uniform granular medium under identical flow conditions [23]. The particle properties of a conventionally uniform medium are defined as mean volume properties. Depending on the properties of the particles, the excess stresses can be either positive or negative. Particles generating positive excess stress during its relaxation move in the direction opposite to lithostatic pressure gradient in gravity flow, i.e. to its free surface. On the contrary, the particles generating negative excess stress move in the direction of lithostatic pressure gradient, i.e. to the bottom of the flow [11].

In formulating the kinetic law of segregation, the magnitude of its flux is defined as the product of velocity coefficient by driving force. As the driving force of the process, the excess moment of gravity, friction and impact forces, which acts on a particle of test component in the gravity flow of nonuniform granular medium [23], is used. The excess moment $\Delta\vec{M}$ is calculated as the difference of total moments of forces acting on a particle of test component in the real flow and a particle of conventionally uniform medium, whose particle properties are defined as mean-volume, under similar flow conditions. The segregation rate coefficient $K_s$ is determined experimentally as the velocity of transverse movement of a test control particle in the gravity flow of a uniform granular medium on a rough chute per unit of the segregation driving force (excess momentum of forces) that induces the particle to move. The magnitude of particle segregation flux of the test component at local values of its concentration and bulk density of the medium is expressed as follows

$$\vec{j}_s = K_s c \rho \Delta\vec{M}. \quad (1)$$

The mechanism of quasi-diffusion separation under conditions of rapid shear deformation of granular media is due to its special state, often referred to as "particulate gas". [6]. In this state mutual displacements of particles of the granular medium acquire a character of quasi-molecular thermal displacements and are accompanied by manifestation of physical effects, the essence of which is formally similar to the diffusion effects revealed in the framework of molecular-kinetic theory [24]. In accordance with this theory, quasi-diffusion of particles, taking place in fast gravity flows of granular media, is accompanied by manifestation of quasi-diffusion effects of both mixing and separation. Quasi-diffusion separation of granular materials is a formal analogue of molecular thermo-barodiffusion. The essence of quasi-diffusion separation is that under the conditions of inhomogeneous dilatancy of the shear flow of granular media, particles with high rates of chaotic movement migrate into sparse flow regions where fluctuations with long free paths are possible. In contrast, particles with properties that limit the rate of their fluctuations concentrate in shear flow regions with high solid phase concentration, in which conditions for chaotic particle movements with short free paths are formed.

Since the dilatancy effect in a granular medium during rapid shear is complexly dependent on the shear rate and lithostatic pressure, when formulating the kinetic law of quasi-diffusion separation (migration) in [7, 14, 16] the relative value of the average distance gradient s between particles

$1/s(\partial s/\partial y)$ was used as the driving force of the process. Rate coefficient $D_m$ of quasi-diffusion separation for binary mixture of spherical particles is expressed analytically, based on the ratio of their quasi-diffusion displacement rates and complex of particle properties influencing the velocity of their fluctuations (diameter $d$, mass $m$, impact recovery factor $k$)

$$D_m = \frac{\overline{m}(c)(\overline{V}')^2}{2F\overline{k}}\left(\frac{d_1^2 k_1}{m_1\overline{d}^2} - \frac{d_2^2 k_2}{m_2\overline{d}^2}\right), \qquad (2)$$

where $F$ is the particle's collision frequency, which is calculated on the basis of the law of conservation of their energy generated by gravity shear, taking into account energy dissipation during collision [7, 25]; $\overline{V}' = F{\cdot}s$ is the mean value of particle's fluctuation velocity; $\overline{d}$ is mean particle diameter; $\overline{m}(c)$ is mean particle mass; $d_i$ is the diameter of the particle of the $i$-th type; $\overline{k}$ is mean value of particle restitution coefficient for nonuniform particles collision [7, 14, 16]; $k_i$ is the impact restitution coefficient for particles of the $i$-th type; $m_i$ is the mass of the $i$-th type particle.

The presented quasi-diffusion separation coefficient reflects the intensity of mutual quasi-diffusion of the components of a binary mixture of particles differing in a set of physical and mechanical properties, depending on their concentration and flow characteristics (shear rate and void volume fraction). The magnitude $j_m$ of quasi-diffusion separation (migration) flux of the test component particles at its local volume concentration $c\rho$ is defined as the product of the driving force by the process kinetic coefficient

$$\vec{j}_m = \rho c D_m \frac{\partial \ln s}{\partial y}. \qquad (3)$$

For two-dimensional steady gravity flow considering separation fluxes (1) and (3), as well as quasi-diffusion mixing flux in transverse direction $y$ and convection transport in shear direction $x$, the general equation of separation dynamics describing the evolution of concentration distribution of test component particles $c(x, y, t)$ is written as follows [7, 14, 16]

$$\frac{\partial(c\rho)}{\partial t} = -\frac{\partial(uc\rho)}{\partial x} + $$

$$+\frac{\partial}{\partial y}\left[\rho\left(D_{\text{dif}}\frac{\partial c}{\partial y} - cD_m\frac{\partial \ln s}{\partial y} - K_{\text{s}}c\Delta M\right)\right], \qquad (4)$$

where $x$, $y$ are coordinates in chute direction and bed height, respectively; $D_{\text{dif}}$ is quasi-diffusion mixing coefficient; $u(y)$ is particle velocity in shear direction; $t$ is time. The quasi-diffusion mixing coefficient of particles is calculated using analogy with molecular kinetics [24]

$$D_{\text{dif}} = \frac{1}{3}s\overline{V}'. \qquad (5)$$

The separation dynamics equation (4) has sufficiently high predictive capability because, except for segregation coefficient $K_{\text{s}}$, all kinetic characteristics included in it are calculated analytically depending on complex properties of binary mixture particles and structural-kinematic parameters of gravity flow. Moreover, the segregation coefficient $K_{\text{s}}$ is determined by direct experimental method [23] and in a wide range of particle properties has features characteristic of the kinetic constant. Predictive capabilities of the equation (4) allow to use it to research the dynamics of the separation process in gravity flow of granular material by computational experiment depending on both particle properties and flow parameters.

The modeling of separation dynamics is carried out by integrating the equation (4) using numerical method. Initial and boundary conditions for the separation dynamics equation (4) are formulated for the case of gravity flow on a rough chute in the following traditional form [7, 14, 16]

$$D_{\text{dif}}\frac{\partial c}{\partial y} = cD_m\frac{\partial \ln s}{\partial y} = K_{\text{s}}c\Delta M\big|_{y=0,h} = 0; \qquad (6)$$

$$c(0,x,y) = c_0, \quad c(t,0,y) = c_0, \qquad (7)$$

where $c_0$ is the average value of concentration of control component in the flow.

The objective of this paper is to investigate physical effects of density separation of impurity particles in a rapid gravity flow of granular material on a rough chute by the method of mathematical modeling of the process dynamics based on the equation (4). The problems of comparative analysis of separation effects' significance and estimation of the degree of tendency of low- and high-density impurity particles separation are solved.

## 2. Materials and Methods

### *2.1. Research method and evaluation of its predictive capability*

In this article a research into the mechanisms and kinetics of the process of particles density separation in gravity flow has been carried out by mathematical modeling method. For this purpose the separation equation (4) with initial and boundary conditions (6) and (7) was solved numerically using implicit Krank-Nikolson difference scheme [26]. To prove the prognostic properties and adequacy of the mathematical model in the modeling of particle density separation, experimental and calculated concentration distributions of less dense particles in a mixture of uniform in size spherical pellets of superphosphate and silica gel in a gravity flow on a rough chute are presented in Fig. 1. During the modeling, the kinetic coefficients of quasi-diffusive separation and mixing were calculated analytically according to the expressions (2) and (5). Segregation kinetics was modeled by analytically determining its driving force – excess moment of forces $\Delta\vec{M}$ depending on properties of interacting particles and shear flow parameters [23]. The experimental value of the segregation coefficient $K_s = 0.6 \cdot 10^3$ $(\text{Ns})^{-1}$ for superphosphate granules as the base component of the mixture [23] obtained in this work was used in the modeling. Experimental distributions of nonuniform particles in the flow on a rough chute and flow parameters required for modeling were obtained in the form of velocity profiles and void volume fraction using experimental-analytical method [23].

The adequacy of the modeling results was established at 95 % confidence level by comparing the variance of adequacy and reproducibility according to Fisher's criterion. In Fig. 1 the results of modeling variants with and without quasi-diffusive separation enable to conclude about the dominant role of the latter in the process of gravity separation of particles with different densities.

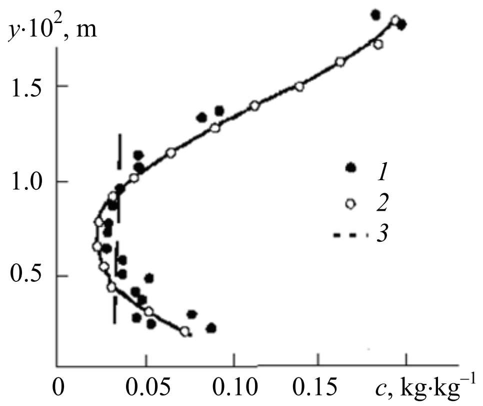


**Fig. 1.** Assessment of adequacy of the mathematical model of separation dynamics by density in the modeling of silica gel particles concentration distributions in gravity flow of a mixture of superphosphate and silica gel particles of the same size: *1* – experimental distributions; *2* – modeling results based on Equation (4); *3* – modeling results based on Equation (4) at $D_m = 0$

### *2.2. Object of study and input data for the modeling*

This paper describes results of the study of the dynamics of particle separation process depending on the relative density of particles in a gravity flow of uniform granular material. The assumption was made that there is no dependence of structural and kinematic parameters of the flow on the relative density of impurity particles because the impurity concentration is small. Structural-kinematic characteristics were determined by experimental-analytical method [23] in the flow of base component particles, on a rough chute. Beads of fraction $(+3.25\text{–}3.5)\cdot 10^{-3}$ m were used as a base component. Structural and kinematic parameters of steady gravity flow of the base component in the form of velocity profiles and free volume fraction are shown in Fig. 2.

During the simulation the impurity content of the granular material remained constant at 12 % mass. The properties of the particles of the impurity

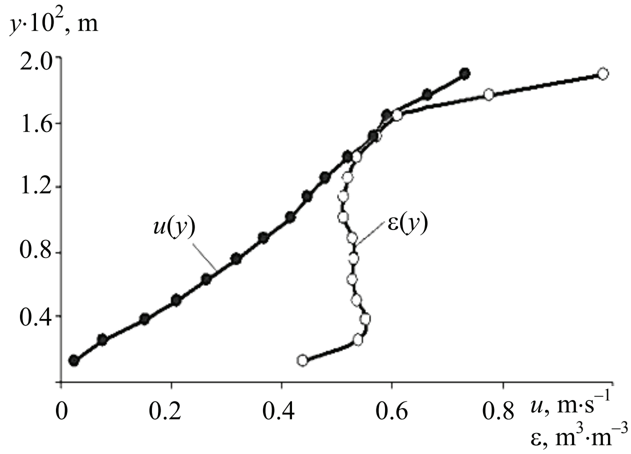


**Fig. 2.** Longitudinal velocity profiles of particles $u(y)$ and void fraction $\varepsilon(y)$ in the gravity flow of glass beads (fraction $(+3.25\text{–}3.5)\cdot 10^{-3}$ m) on a rough chute

(control component) were assumed to be identical to those of the particles of the base component, except for density. The density of the impurity particles $\rho_t$ was varied with respect to the particle density of the base component $\rho_b$ in the range of values $\rho_t / \rho_b = 0.2 - 4.0$ . All kinetic parameters included in Equation (4), except for segregation coefficient $K_s$, were determined analytically as functions of particle properties, their local concentrations and flow characteristics. The segregation coefficient was taken equal to $1.8 \cdot 10^3$ $(\text{Ns})^{-1}$, according to the results of the study presented in [23] for bead pellets as the base component of the mixture.

## 3. Results and Discussion

### *3.1. Dynamics and separation effects of high and low density particles*

As examples of the obtained modeling results which confirm the dominant role of quasi-diffusion separation effect in gravity flow of nonuniform in density particles, concentration distributions of the test component (impurity) particles by bed height are shown in Fig. 3.

The modeling results are given for the cases when relative density of the control particles is less than 1 (Fig. 3*a*) and more than 1 (Fig. 3*b*). In order to analyse the significance of segregation and quasi-diffusion effects in the formation of impurity particle distribution in the gravity flow, modeling was carried out according to variants which differed from each other either by complex consideration of the specified separation effects, or by alternative consideration of them. Analysis of the given results shows that in the density separation process the quasi-diffusion separation effect clearly dominates in cases of both low and high density of impurity particles. It is important to note that the low density particle impurity distribution (Fig. 3*a*) has a shape similar to the shape of the void volume fraction profile in the flow (see Fig. 2), while the shape of the concentration distribution of high density impurity particles (Fig. 3*b*) obviously corresponds to the shape of the volume fraction distribution of the solid phase. This alternative distribution of low- and high-density impurity particle in the gravity flow indicates a pattern of high fluctuation rate particles moving into the high void volume fraction flow area while simultaneously shifting low fluctuation rate particles into the area with high solid volume fraction flow.

In order to assess the degree of separation effects depending on the density of the mixture components, for each variant of the density ratio of the impurity particles and the base component the concentration distribution of the test component $c(y)$ in the steady-state phase of the process was determined. The obtained concentration distributions of the mixture were used to estimate its heterogeneity over the bed height and, accordingly, the degree of separation effects. The coefficient of variation was used as a characteristic of distribution heterogeneity, which was calculated based on a layer-by-layer analysis of the concentration of test particles, taking into account the relative mass fraction of

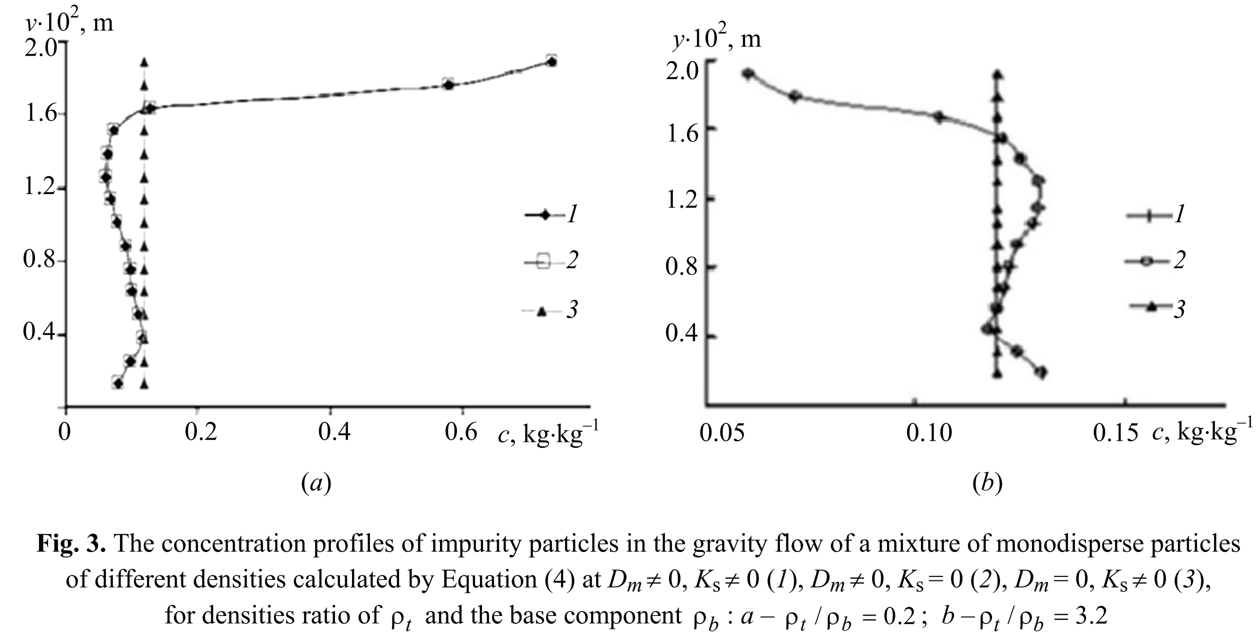


**Fig. 3.** The concentration profiles of impurity particles in the gravity flow of a mixture of monodisperse particles of different densities calculated by Equation (4) at $D_m \neq 0$, $K_s \neq 0$ (*1*), $D_m \neq 0$, $K_s = 0$ (*2*), $D_m = 0$, $K_s \neq 0$ (*3*), for densities ratio of $\rho_t$ and the base component $\rho_b$ : $a - \rho_t / \rho_b = 0.2$ ; $b - \rho_t / \rho_b = 3.2$

elementary layers in the flow at the chute, according to the following expression

$$V = \frac{100}{\bar{c}} \sqrt{\frac{\sum u_i (1-\varepsilon_i)\rho_i(c)\Delta H_i (c_i - \bar{c})^2}{\sum u_i (1-\varepsilon_i)\rho_i(c)\Delta H_i}}, \quad (8)$$

where $\bar{c}$ is the average concentration of impurity particles in the flow, kg·kg$^{-1}$; $c_i$ is the average concentration of impurity particles in the $i$-th sub-layer of the flow, kg·kg$^{-1}$; $u_i$ is average flow velocity in the $i$-th sub-layer, m·s$^{-1}$; $\varepsilon_i$ is the average value of void volume fraction in the $i$-th sub-layer, m$^3$·m$^{-3}$; $\rho_i$ is average particle density in the $i$-th sub-layer, kg·m$^{-3}$; $\Delta H_i$ is the thickness of the $i$-th sub-layer, m.

Figure 4 shows the dependence, reflecting the influence of the relative density of impurity particles on the inhomogeneity of its distribution in the gravity flow under the effects of separation. When analyzing the obtained dependence, special attention is paid to the clearly higher intensity of the particle density separation process of impurity particles in cases where their density is less than the density of the base component particles ($\rho_t / \rho_b < 1$), as compared with alternative values of density ($\rho_t / \rho_b > 1$). A fundamental difference is also observed in the dynamics of the increase of the variation coefficient with the increase of the degree of difference between the density of the impurity particles and the density of the base component particles in the cases $\rho_t / \rho_b < 1$ and $\rho_t / \rho_b > 1$. With the increasing degree of difference in the density of the base component particles and the impurity in the region of high values of the density ratio ($\rho_t / \rho_b > 1$), the coefficient of variation changes at a decreasing rate. In contrast, for impurity particles whose density is lower than that of the base component particles ($\rho_t / \rho_b < 1$), the rate of increase of the variation coefficient does not slow down with the increasing degree of difference between the particles.

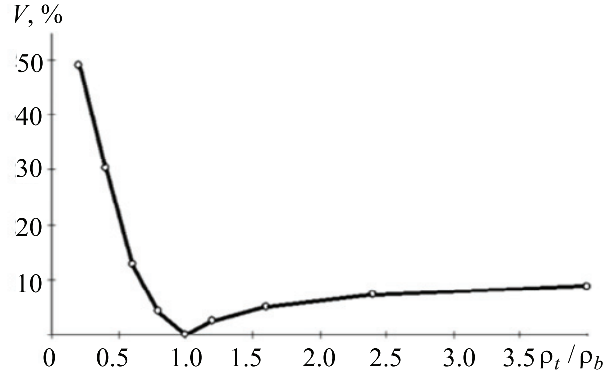


**Fig. 4.** Variation coefficient of the concentration distribution of impurity particles in the gravity flow of monodisperse granular material as a function of the ratio of the density of impurity particles to the density of the base component particles

### *3.2. Analysis of kinetic regularities of quasi-diffusion separation effect for high and low density particles*

In order to explain the observed features of the dependence $V = f(\rho_t / \rho_b)$, taking into account the dominant role of the quasi-diffusion separation effect (see Fig. 3), the analysis of kinetic regularities of the effect in relation to its driving force and drag in the gravity flow of particles with different densities was carried out [27]. The analysis is based on the basic provisions of the kinetic theory, the possibility of adaptation of which to describe the dynamics of fast gravity flows of cohesionless granular materials is confirmed by numerous studies [6, 7, 16, 28–32].

Taking into account the peculiarities of modeling the separation process of particles differing only in density, we have a case of equality of diameters ($d_1 = d_2 = \bar{d}$) and restitution coefficients ($k_1 = k_2 = \bar{k}$) at collision of uniform particles. In this case, according to the given dependence (2) for $D_m$, the value of quasi-diffusion separation coefficient (migration) for particle mixture of test and base components will be determined by the following expression

$$D_m = \frac{\rho(c)(\overline{V'})^2}{2F}\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right), \quad (9)$$

where $\rho(c) = c\rho_t + (1-c)\rho_b$ is the average density of particles in the mixture.

In invariant flow conditions (see Fig. 2) and at low concentrations of impurity it is possible to assume that the dependence of the complex $\frac{(\overline{V'})^2}{F}$ on the density of the test component is negligibly small. Therefore, the value of the quasi-diffusive separation coefficient (mutual quasi-diffusion of particles) in first approximation is proportional to the complex $\rho(c)\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$. Since in different variants of modeling for uniform in size particles the dynamics of changes in mean spacing along the bed height remains the same, it is obvious that the virtual experiment is carried out at a constant value of the driving force of quasi-diffusion separation $1/s(\partial s/\partial y)$.

Then, taking into account the dominant role of quasi-diffusion separation effect (see Fig. 3) it can be stated that the efficiency of the separation process in modeling variants will change in proportion to the complex $\rho(c)\left(\frac{1}{\rho_1}-\frac{1}{\rho_2}\right)$. The analysis of the complex shows, that depending on the value of the ratio of particle densities of test and base components $\rho_t/\rho_b$ the complex will have either positive (at $\rho_t/\rho_b \le 1$), or negative (at $\rho_t/\rho_b > 1$) values. And the sign of the complex indicates the direction of migration of the control component particles, while the modulus of the complex determines the intensity of quasi-diffusion separation flow. At a positive value of the complex the direction of test component particles migration coincides with the direction of the average void volume, while at a negative value it is opposite to the indicated gradient. Figure 5 gives the dependence of the complex $\rho(c)\left(\rho_t^{-1}-\rho_b^{-1}\right)$ on the migration intensity and its absolute value on the ratio $\rho_t/\rho_b$. The dependence is obtained for a fixed value of the test component concentration ($c = 0.12$). When comparing the dependencies shown in Figures 4 and 5, it can be seen that there is an extremely high degree of similarity between them. Taking into consideration that a variation coefficient and the complex $\rho(c)\left(\rho_t^{-1}-\rho_b^{-1}\right)$ of the same argument ($\rho_t/\rho_b$) are plotted, one can argue that this complex plays a crucial role not only in the kinetics of the quasi-diffusive separation effect. It is also quite obvious the paramount importance of the complex in estimating the tendency of impurity particles of different density to separate in fast gravity rarefied flow of granular material.

In addition, the analysis of dependencies presented in Figs 4 and 5, with regard to the concentration distribution profiles of test particles of different densities (Fig. 3) and stable values of the driving force of the separation process allows to draw the following conclusion. A mixture of monodisperse particles of different densities is characterized by the highest propensity to separation in the gravity flow, if it contains a relatively small volume admixture of particles with low density. In this case low density particles acquire in shear flow the highest rate of fluctuations and, as a consequence, high rate of quasi-diffusion percolation of the granular medium [27]. Obviously, the particles with low density will be characterized by the highest velocity of quasi-diffusion percolation of the granular medium at their concentration in the flow $c \to 0$.

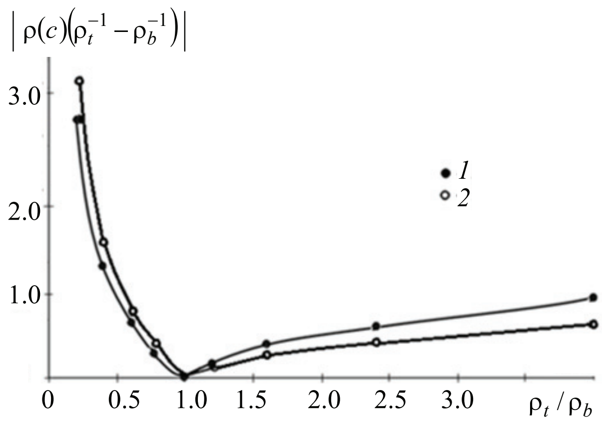


**Fig. 5.** Analysis of the kinetics of quasi-diffusion separation on density of impurity particles in gravity flow depending on their relative density at different concentrations in the flow: *1* – $c = 0.12$; *2* – $c \to 0$

On the contrary, with the decrease in the concentration of impurity particles with high density, average velocity of their quasi-diffusion displacement in shear flow decreases, due to smaller mean value of impulses affecting the test dense particle in the flow and reduction of its fluctuation velocity. As a consequence, the value of quasi-diffusion separation coefficient for the dense particles impurities is less than its value for the low-density particles impurities, provided that their concentration and density ratios are equal.

The stated physical conceptions in relation to the influence of the concentration of impurity particles, different in density from the base component particles, on the intensity of their separation in gravity flow are confirmed by the comparison of the dependencies shown in Fig. 5. Dependences of the complex value $\rho(c)\left(\rho_t^{-1}-\rho_b^{-1}\right)$ on the relative density of particles for two values of impurity concentration indicate an increase in the intensity of separation of impurity particles with low density, as well as the lower intensity of separation of dense impurity particles with a decrease in their concentration in the flow.

## 4. Conclusion

Using a mathematical model which describes the dynamics of distribution of nonuniform particles in a rapid gravity flow of granular material, the efficiency of the separation process of impurity particles of different densities has been investigated. The mathematical model reflects the kinetics of convection, mixing, segregation and quasi-diffusion separation effects of particles of the test component with respect to a set of physical and mechanical properties. In the process of modeling, distribution profiles of the impurity particle concentration in the flow on a rough chute were obtained, the relative density of the particles varied in the range of values 0.2–4.0 with respect to the density of the monodisperse particles of the base component. The distribution profiles confirm the determining role of the quasi-diffusion separation effect in their formation.

It is found that the low-density impurity particles distribution has a shape similar to the shape of the void volume fraction profile, while the shape of the concentration distribution of high-density impurity particles corresponds to the shape of the solid phase volume fraction distribution. This distribution of low- and high-density particles is explained by the fact that particles with high fluctuation velocity move into the high void volume fraction flow area, while low fluctuation velocity particles move into the high solid volume fraction flow area.

The higher intensity of separation of impurity particles with lower density than that of the base component was found in comparison with the variants with high density of impurity particles. This is explained by the increase of diffusive permeability of impurity particles with the decrease of their density as a result of the growing fluctuation velocity. The increase of diffusive permeability for low-density impurity particles leads to higher mutual quasi-diffusion coefficient (quasi-diffusion separation) and intensity of the process.

A fundamental difference was found in the dynamics of increasing coefficient of variation in the distribution of high and low density impurity particles when their difference from the base component particles was growing. As the degree of difference between the high-density particles and the base component increases, the coefficient of variation changes with a decreasing rate. In contrast, for impurity particles whose density is lower than that of the base component particles, the rate of increase in the variation coefficient does not slow down with increasing degree of difference between the particles.

It is found that as the concentration of impurity particles in the flow reduces, the intensity of their separation increases for particles with low density and decreases for particles with high density. This is explained by the fact that as the impurity concentration goes down, the average fluctuation velocity of low density impurity particles increases and the fluctuation velocity of high density impurity particles decreases. The increase in average fluctuation velocity of low-density impurity particles is a consequence of the increase in the average integral value of shock pulse acting on it with rising concentration of denser particles of the base component in the flow. Correspondingly, the decrease of average fluctuation velocity of high-density impurity particles is caused by the decrease of average integral momentum value, perceived by such particles, with the increase of concentration of low-density particles in the flow.

The results of the study suggest that the highest propensity to density separation of particles in the rapid gravity flow of granular material is characterized by a binary mixture of particles of uniform size with a minimum volume content of impurity particles with low density.

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## 6. Conflict of interests

The authors declare no conflict of interest.

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