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Phenomenology of clustering and separation effects in granular media under vibration impact in microgravity conditions

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Abstract: The paper provides the phenomenological description of the effects of cluster formation and separation of cohesionless spherical particles by size and density in a bed of a rarefied granular medium under the action of vibrations in microgravity conditions. Mathematical modeling of the structural and kinematic parameters of the granular medium is performed based on its equation of state as a "gas of solid particles". It is established that the condition for particle cluster formation is sufficiently high values of the solid phase fraction and the thickness of the vibrating bed, at which the quasi-thermal vibration flux has a limited area of active penetration into the bed volume. The separation process is a consequence of the quasi-diffusion interaction of particles with different fluctuation velocity in the presence of a gradient in the fraction of voids in the granular medium. The distribution of particles of a binary mixture of varying sizes was simulated using the separation dynamics equation, which describes the transport of non-uniform particles as a result of the coupling of quasi-diffusion separation and mixing fluxes. The simulation results are compared with experimental data obtained with support from the European Space Agency (Parabolic Flight Campaign PFC64) using the VIP-Gran instrument.

Keywords: granular material; microgravity; vibration; clustering; quasi-diffusion separation by size and mass; mixing; granular temperature.

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Феноменология эффектов кластеризации и сепарации в зернистых средах при вибрационном воздействии в условиях микрогравитации

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Аннотация: Предложено феноменологическое описание эффектов формирования кластеров и сепарации несвязных сферических частиц по размеру и плотности в слое разреженной зернистой среды под действием вибраций в условиях микрогравитации. Проведено математическое моделирование структурно-кинематических параметров зернистой среды на базе уравнения ее состояния как «газа твердых частиц», в результате которого установлено, что условием формирования кластера частиц являются достаточно высокие значения доли твердой фазы и толщины вибрирующего слоя, при которых квазитепловой поток вибрации имеет ограниченную область активного проникновения в объем слоя. Процесс сепарации является следствием квазидиффузионного взаимодействия частиц, имеющих различную скорость флуктуаций при наличии градиента доли пустот в объеме зернистой среды. Моделирование распределения частиц бинарной смеси, различающихся по размеру, проведено с использованием уравнения динамики сепарации, описывающего процесс переноса неоднородных частиц как результат сопряжения квазидиффузионных потоков сепарации и перемешивания. Результаты моделирования приведены в сравнении с экспериментальными данными, полученными при поддержке европейского космического агентства (Parabolic Flight Campaign PFC64) с использованием прибора VIP-Gran.

Ключевые слова: зернистый материал; микрогравитация; вибрация; кластеризация; квазидиффузионная сепарация по размеру и массе; перемешивание; температура зернистой среды.

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1. Introduction

The study of small celestial bodies (asteroids and planetoids) in the Solar System is of continuing interest, both from scientific and practical perspectives [1–5]. On the one hand, the research results allow us to come closer to explaining the mechanisms underlying the origin of the planetary system [2–7] and life on Earth, and, on the other hand, to predict ways to expand the existing resource base with sources of rare earth elements and water. A comprehensive overview of the interdisciplinary findings on various aspects of small celestial bodies, conducted using analytical and experimental methods, including DEM modeling, various types of astronomical observations, and space missions, is presented in a review paper prepared by a large international team of authors [1]. A significant portion of this paper is devoted to the analysis of these celestial bodies as granular systems from the standpoint of classical mechanics and condensed matter physics, with the goal of identifying fundamental differences in their behavior under microgravity conditions.

In this regard, the behavior of non-uniform granular materials under conditions of minimal gravitational influence has attracted increasing attention over time [8–13]. The relevance of this research is confirmed by the expanding range of cognitive and practical problems associated with the exploration and development of near and deep space. In near-space environments, granular media, such as the regolith covering the surface of the Moon and hundreds of thousands of planetoids in the Solar System, can serve as a building material for space constructions and, in addition, solve the problem of a shortage of unique mineral raw materials [1, 2].

Moreover, microgravity conditions appear to be extremely favorable for fundamental research into the effects of particle interactions in granular materials and their physical and mechanical properties [11], such as wave effects and the dissipative properties of materials [14, 15], problems of gas-liquid quasi-interphase transitions [11, 16], and fundamental problems in the physics of nonequilibrium states of granular media [17, 18].

This study was initiated by the results of physical and virtual experiments with granular materials consisting of spherical, cohesionless, not entirely elastic, and non-smooth particles under microgravity and vibration conditions [2]. The experiments revealed pronounced clustering effects in the granular material and identified certain patterns in the formation of heterogeneous structures. According to the authors of [2], the identified cluster structures are characterized by a non-uniform distribution of both the solid phase and non-uniform particles. They appear largely enigmatic in the absence of gravitational influence and require an explanation of the phenomenology of their physical mechanisms. The present work aims to provide a phenomenological explanation and develop a method for predicting the effects of cluster formation and non-uniform particle distributions under vibration influence on a granular medium under microgravity conditions.

2. Materials and Methods

2.1. The object of the study and its analysis

By analogy with work [2], monodisperse bronze particles and their binary mixtures with different component concentrations were used as model materials in this research. The sufficiently large particle size of the components (1 and 2 mm) allows us to characterize the model materials as non-cohesive granular media. The basic experimental information in this study was the results of physical and virtual experiments performed within the framework of work [2]. The virtual experiment was performed by simulating the displacement of particles as a result of their contact interactions during vibration oscillations using the finite element method (DEM). Under conditions of a low volume fraction of the solid phase (0.03–0.17), particle contacts are limited by their impact interactions, which are accompanied by the action of frictional forces and impact pulses. The frictional and impact pulse forces are determined in proportion to the tangential and normal components of the relative velocity of the colliding particles, respectively.

The DEM modeling results are in good agreement with the results of a physical experiment supported by the European Space Agency (Parabolic Flight Campaign PFC64) using the VIP-Gran instrument. The instrument consists of a measuring cell loaded with granular material – a rectangular container (45 × 30 × 5 mm). The opposite sides of this container measuring 30 × 5 mm function as pistons and oscillate harmonically in antiphase with a frequency of 20 Hz and an amplitude of 3 mm.

The experiments revealed that at a small cell load of large particles (80 pcs.) with a solids fraction of $1 - \varepsilon = 0.05$ (the solids fraction is defined by the authors of this paper as the ratio of the particle volume to the cell volume) the granular medium (sample 1) behaves like a gas of solid particles. This is confirmed by the chaotic movement of the particles and, according to the authors [2], their uniform distribution throughout the volume. When the cell loading is increased to a volume fraction of $1 - \varepsilon = 0.089$, by adding a corresponding number of small particles (500 pcs.) to the cell containing large particles, a heterogeneous structure is formed in the granular medium (sample 2), with a cluster in the central zone of the cell having a high concentration of the solid phase and large particles. Conversely, the cell regions located near the vibrating surfaces are characterized by a low solid phase content and a high concentration of small particles. An increase in the volume fraction of the solid phase and the concentration of large particles in the cell leads to the cluster volume in its central zone increasing practically proportionally to the volume of the added particles. It is important to note that, according to visual analysis of the experimental data, in cells loaded with non-uniform particles, the zones bordering the vibrating surfaces contain exclusively small particles in a gaseous state of solid particles with an approximately equal distribution of their concentration in the direction of vibration.

2.2. Selection of research method and object of the study

The shortcomings of traditional methods of mathematical modeling of granular media dynamics based on finite element analysis (DEM) [19, 20] and the continuum approach [21, 22] fundamentally limit their predictive capabilities and are a factor determining the need to develop phenomenological models [20, 23]. According to [19, 24], phenomenological models of granular media dynamics require intensive development.

This paper develops a phenomenological model of the dynamics of the structural and kinematic parameters of a granular medium and the distribution of its non-uniform components under microgravity conditions and vibration action on it from the bounding surfaces [2]. The bounding surfaces are formed by two parallel plates, the dimensions of which, compared to the distance between them, are so large that the volume of the medium between them can be represented as an infinite plate. The surfaces perform harmonic oscillations in antiphase with complete coincidence of the vibration parameters. The listed characteristics of the object allow us to represent the problem being solved as one-dimensional and symmetrical.

The proposed phenomenological description establishes a relationship between the parameters of vibrational oscillations and a set of particle characteristics of a granular medium, with its parameters determining the conditions of dynamic particle interaction. This description is intended for use in predicting the distribution of particle fluctuation velocities and the fraction of voids (porosity) within a granular medium subjected to vibrational oscillations in the absence of gravitational influence, depending on the vibration parameters, the complex of physical and mechanical properties of the particles, and the volume fraction of the solid phase.

The description of the relationship between the structural and kinematic characteristics of a granular medium is based on the physical analogy of a granular medium in a rarefied state under conditions of weightlessness and vibrational oscillations with gas dynamics [25, 26]. The physical analogy suggests the existence of a relationship between the dilatancy of a granular medium and its pressure and granular temperature (the kinetic energy of the relative movements of particles) with a formal similarity of the relationship between the named parameters with the equation of gas dynamics:

$$p\bar{\varepsilon}(y) = \chi\theta(y), \qquad (1)$$

where p is the pressure of a granular medium (dispersion pressure according to Bagnold [27]) due to fluctuations of its particles under the action of vibrational oscillations; $\theta(y)$ is the local temperature value of the granular medium (kinetic energy of chaotic movements of particles); $\bar{\varepsilon}(y)$ is the dilatancy of the medium caused by the quasi-thermal movement of particles; χ is the coefficient of the state equation of state; y is the coordinate in the direction of vibrational oscillations.

Dilatancy of a granular medium is defined by the expression [26]

$$\bar{\varepsilon}(y) = \frac{\varepsilon(y) - \varepsilon_0}{1 - \varepsilon(y)}, \qquad (2)$$

where ε_0 is the volume fraction of voids in the granular medium under stationary bed conditions, i.e., at $\theta(y) = 0$; $\varepsilon(y)$ is the local value of the fraction of voids in the volume of the medium at the nominal dispersion pressure p and granular temperature $\theta(y)$ caused by vibration action.

In defining a formal analogy between the state of a granular medium under intense relative motion of its particles and gas dynamics, the granular temperature parameter is of primary importance. In the traditional representation, temperature is expressed [28] as a parameter proportional to the averaged instantaneous value of the squared component of the particle velocity of chaotic fluctuations. In this paper, the granular temperature is represented as the kinetic energy of particles, caused by their relative motion during fluctuations during mutual collisions under the influence of vibrational oscillations [25, 26]. It should be noted that the definition of temperature as the kinetic energy of particles is used in a number of other studies [29, 30].

According to equation (1), its right-hand side expresses the kinetic energy of particles, which they possess due to their relative motion under the influence of vibrational oscillations. The left-hand side of the equation reflects the work performed by particles per unit volume of the solid phase, accompanied by the dilatancy effect, under the influence of vibrational oscillations. Consequently, the coefficient χ of the equation can be recognized as a parameter defining the relationship between the work performed by particles under the influence of vibrational oscillations, accompanied by the dilatancy effect, and the kinetic energy of the relative displacements of the particles.

Equation (1) is used to analyze the quasi-thermal effect of vibrational oscillations and the corresponding quasi-thermal flux in relation to their influence on the structural and kinematic characteristics of a granular medium and the kinetics of the mixing and separation processes of non-uniform particles. The quasi-thermal effect of vibrational action is estimated [25, 26] by local values of the kinetic energy of the particles (the temperature of the granular medium) under the assumption of the dominant role of the kinetic energy of chaotic particle fluctuations, neglecting the energy of their rotation around their own axes. This assumption is supported by the results of a study [31], according to which the role of the rotational energy of particles becomes significant only under conditions of high concentrations of "solid particle gas".

The component of the kinetic energy of particles with non-uniform density, generated by vibration and caused by the presence of a randomly distributed fluctuation velocity in the solid phase elements, is calculated as [25, 26]

$$\theta = \frac{1}{2}\rho(y)\langle V'(y)\rangle^2, \qquad (3)$$

where $\rho(y)$ is the average particle density; $\langle V'(y)\rangle$ is the average absolute value of the instantaneous fluctuation velocity of conventionally uniform particles, which will be defined below as a function of the coordinate depending on the vibration parameters, the physical and mechanical properties of the particles, and the structural characteristics of the medium.

2.3. Evolution of the structural and kinematic characteristics of a granular medium under vibrational oscillations in the absence of gravity

The temperature of a granular medium θ is determined under the assumption of its properties. The granular medium is considered as a set of spherical non-smooth inelastic particles. During vibrational oscillations of the particles in the absence of gravity, the set of particles is in a state similar in physical properties to an elastic, compressible body. The validity of this assumption is substantiated by the mesoscopic properties of granular materials, according to which individual particles exhibit the properties of a solid, while a given set of particles, depending on the conditions of their interaction, can exhibit properties similar to those of substances in the liquid and gaseous states [32]. Harmonic oscillations generated in the medium by the surface bounding its volume will be accompanied by successive localized decreases and increases in volume with a frequency corresponding to the frequency of the vibrational oscillations. Successive compressions and increases in the volume of a medium are associated with the effects of supply and dissipation of quasi-thermal energy from vibrational oscillations [26].

The quasi-thermal flux $Q(y)$ generated by vibrating surfaces is expressed in accordance with the diagram shown in Fig. 1. The quasi-thermal flux emanating from the vibrating surfaces penetrates the granular medium bed, being directed opposite the

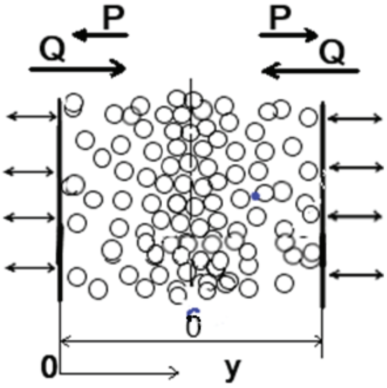

Fig. 1. Schematic diagram of quasi-thermal flux generation by vibrating surfaces in a granular medium in the absence of gravity

dispersion pressure vector p, and loses its intensity due to energy dissipation during the collision of non-smooth, inelastic particles. Quasi-thermal fluxes will be maximum near vibrating surfaces ($y = 0$), where the flux intensity is determined by the dispersion pressure of the medium and the characteristics of the vibrational oscillations.

In the presence of dissipation effects, contact between particles and the vibration source is systematically interrupted. The moment of interruption of contact between particles and the surface corresponds to the state of ultimate compression of the medium, when the surface coordinate is at its maximum. In this case, the moments of maximum compression of the granular medium will be determined as follows:

$$t_0 = \frac{n - 1 + 0.25}{\omega}, \tag{4}$$

where n is the ordinal number and ω is the frequency of the vibrational oscillations.

At the moment of loss of contact with the surface, the medium transitions from a state of ultimate compression to a state of dilatancy. The intensity of dilatancy is determined by the velocity of the quasi-diffusion displacements of the particles. The contact of particles with the surface is resumed in the next period of oscillation in the collision mode during counter-movement. The coordinate of contact resumption and the corresponding time t_c in the oscillation period are determined by solving a system of equations describing the surface displacements in its harmonic oscillations y_v and the boundary y_m of the increasing volume of the medium in accordance with the quasi-diffusion mechanism. Assuming a small relative fraction of the volume of the medium in which contact occurs, the coordinate of contact resumption is determined by solving the system of equations:

$$y_m = A - \frac{1}{3}\langle V'(0)\rangle (t - t_0), \tag{5}$$

$$y_v = A\sin(2\pi\omega t), \tag{6}$$

where A is the amplitude of the vibrational oscillations ($A << \delta/2$), and $1/3\langle V'(0)\rangle$, and denotes the average value of the y – component of the particle fluctuation velocity in the granular medium adjacent to the vibrating surface.

The quasi-thermal flux generated in the granular medium by the vibrating surface can be expressed in the following integral formulation [25, 26]:

$$Q(0) = \int_{t_c}^{t_0} \omega p_y(0)\upsilon(t)dt, \tag{7}$$

where the parameter $\upsilon(t) = 2\pi A\omega\cos(2\pi\omega t)$ determines the magnitude of the vibrating surface velocity during the oscillation period.

The dispersion pressure p_y, caused by particle fluctuations, is determined under the assumption that it is uniform in a limited volume of a rarefied granular medium in the absence of factors limiting the relaxation of internal stresses. In this case, the pressure within the volume of the granular medium can be assumed to be equal to its pressure on the vibrating surface. The magnitude of this pressure is expressed as a function of the intensity of the impulse action exerted on the granular medium by the vibrating surface. It is assumed that the change in the impulse of particles in contact with the vibrating surface during the oscillation period $1/\omega$ is equal to the impulse of the force acting on the surface. The analysis is performed under the assumption that contact between the particles and the surface occurs in a boundary bed which thickness is much smaller than the thickness of the bed of the medium enclosed between the vibrating surfaces. According to the findings in [11], this assumption ensures a stable condition for the formation of a state of the granular medium corresponding to a “gas of solid particles”.

Under this condition, the mass of particles in the boundary bed of the medium $\Delta\overline{m}(0)$ coming into contact with the vibrating surface per unit area during the oscillation period can be expressed as the product of the volume concentration of the solid phase in the boundary bed, the velocity of its quasi-diffusion movement in the direction of the vibrating surface, and the oscillation period.

$$\Delta m(0) = \overline{m}(0)(b(0)d(0))^{-3}\frac{1}{3}\langle V'(0)\rangle / \omega, \quad (8)$$

where $\overline{m}(0)$ is the average value of the mass of a particle of a uniform granular medium in the boundary bed; $(b(0)d(0))^{-3}$ is the volume concentration of the number of particles in the boundary bed of the medium, where $b(y) = [\pi/(6(1-\varepsilon(y)))]^{0.33}$ is a geometric parameter determined depending on the local value of the void fraction $\varepsilon(y)$.

The change in the momentum of boundary bed particles is determined in accordance with the impact mechanism of their interaction with a vibrating surface. As a result of collision with the surface, particles experience an impact momentum proportional to the relative velocity of the colliding particles and the surface along the impact line. The relative velocity V_r is defined as the sum of the average values of the opposing moduli of the pre-impact velocity vectors of the normal *y* component of the quasi-diffusion flux of particles of a rarefied granular medium and a vibrating surface.

$$V_r = \frac{1}{3}\langle V'(0)\rangle + \frac{1}{t_0 - t_c}\int_{t_c}^{t_0} 2\pi A\omega\cos(2\pi\omega t)dt. \quad (9)$$

Taking into account energy dissipation due to elastic deformations and friction [25, 26], the change in particle momentum upon collision with a unit area of the surface over one period of its oscillation is determined by the expression

$$\Delta m\Delta V_r = \Delta m(0)V_r(1+(1-k_E))^{0.5}, \quad (10)$$

where k_E is a parameter defining the average integral value of the portion of the particle kinetic energy that is dissipated during their collision in the oblique impact mode. The value of the parameter k_E is calculated based on a hypothesis that accounts for the specific features of head-on and sliding collisions of spherical particles by combining the Newtonian and Routhian hypotheses [25, 26, 33]. To calculate the parameter k_E, the coefficients of restitution *k*, friction μ, and the reduction coefficient of the tangential velocity component λ during particle collisions are required in the following form:

$$k_E = (1-k^2) + \frac{1}{2}\lambda - \frac{1}{8}\mu^2(1+k)^2 - \frac{1}{8}\lambda^2 + \frac{2}{\pi}\mu(1+k) - \frac{2}{3\pi}\mu\lambda(1+k). \quad (11)$$

The reduction coefficient of the tangential velocity component during particle collisions λ is determined using the method proposed in [33]. Thus, the difference $1-k_E$ determines the magnitude of the modified coefficient of restitution, which takes into account the effects of elasticity and roughness, during the collision of non-smooth inelastic balls under conditions unregulated by the collision angle.

Since the change in momentum is equal to the momentum of the force causing this change, the average value of the pressure of the granular medium on the vibrating surface can be expressed as follows:

$$p_y = \Delta m(0)V_r(1+(1-k_E))^{0.5}\omega. \quad (12)$$

Thus, using expression (7) in combination with the last relation, it is possible to calculate the intensity of the quasi-thermal flux $Q(0)$ generated by a vibrating surface in the boundary bed of a rarefied granular medium. The main problem in implementing this approach is determining the average value of the particle fluctuation velocity in the boundary bed of a granular medium $\langle V'(0)\rangle$. In general, the fluctuation velocity should be determined [25, 34] based on the dissipative component of the energy balance for an element of the granular medium as a function of its structural characteristics, the parameters of the medium filling the space between the particles, and the properties of the particles. In this paper, the fluctuation velocity is determined by the temperature of the granular medium (3) as a function of the intensity of the quasi-thermal vibration flux [25, 26].

To determine the magnitude of the quasi-thermal flux of vibrational oscillations in the volume of a granular medium, the postulate [25, 26] was used, according to which the magnitude of the flux is calculated in direct dependence on the local values of the temperature of the granular medium $\theta(y)$ and the concentration of particles per unit surface area located in a plane perpendicular to the quasi-thermal flux $(b^{-2}(y)d^{-2}(y))$, and the frequency ω of the vibrational oscillations:

$$Q(y) = b^{-2}(y)d^{-2}(y)\theta(y)\omega. \quad (13)$$

Due to the dissipation of energy during collisions, the magnitude of the quasi-thermal flux decreases during its penetration into the volume of the granular medium. As a result, the dynamics of the quasi-thermal flux of vibrational oscillations in a particle bed can be expressed as follows:

$$Q(y) = Q(0) - \int_0^y \Delta Q(t)\,dt, \quad (14)$$

where $\Delta Q(y)$ is a parameter that determines the amount of energy dissipated per unit volume of the medium.

With insignificant resistance of the medium in the space between particles, the calculation of $\Delta Q(y)$ can only be performed taking into account the energy dissipated during elastic deformation and friction of the particles (11). In this case, based on postulate (13) and taking into account the energy dissipation flux, which is determined proportionally to the local value of the volume concentration of particles $(b(y)d(y))^{-3}$, a relationship has been formulated in [25, 26] that allows one to determine the intensity of the quasi-thermal flux of vibrations depending on the depth of its penetration into the volume of the medium

$$Q(y) = Q(0)e^{-\int_0^y k_E (b(t)d(t))^{-1} dt}. \quad (15)$$

Dependence (15) reflects the influence of the structural heterogeneity of a granular medium on the attenuation intensity of the quasi-thermal flux of vibrations and, according to Expression (11) of the physical and mechanical properties of the particles.

Given the known intensity of the local values of the quasi-thermal flux $Q(y)$, taking into account its relationship with the granular temperature in accordance with postulate (13), the temperature can be expressed as a function of the coordinate in the direction of the flux:

$$\theta(y) = Q(y)(b(y)d(y))^2 / \omega. \quad (16)$$

Thus, to predict the local values of the quasi-thermal flux (15) and the granular temperature (16), it is necessary to have local values of the parameter $b(y) = [\pi/(6(1-\varepsilon(y)))]^{0.33}$, i.e. the geometric characteristic of the structure, and the average particle diameter

$$d(y) = (c(y)/\rho_1 d_1 + \\ + (1-c(y))/\rho_2 d_2)(c(y)/\rho_1 + (1-c(y))/\rho_2)^{-1}.$$

Determining these parameters requires knowledge of the local values of the void fraction $\varepsilon(y)$ and the concentration of nonuniform particles $c(y)$ in the volume of the granular medium.

In turn, determining the distribution of the volume fraction of voids $\varepsilon(y)$ using the equation of state (1) presupposes the presence of a temperature distribution profile of the granular medium $\theta(y)$. The distribution profiles of the physical parameters of the granular medium ($\varepsilon(y)$, $\theta(y)$) are formed during the evolution of the initial homogeneous distributions during successive periods of oscillation. To determine the profiles, their evolution is modeled during a successive transition from one profile to another and from period to period until the medium parameters in the subsequent period differ negligibly from the parameters in the previous period. To implement the algorithm in the first period of vibration oscillations, the quasi-heat flux $Q_1(y)$ and temperature profile $\theta_1(y)$ are determined for an initial homogeneous distribution of particles in the medium in the absence of chaotic movements $(V'(y) = 0)$ as follows. Using equation (7), with an initial homogeneous distribution of motionless particles in the medium, the intensity of the quasi-thermal flux in the boundary volume of the bed $Q_1(0)$ is determined, as well as the flux dynamics in the bed using (15) and the granular temperature distribution in the medium (16). The dispersion pressure is determined based on the fact that the mass of particles that will contact the vibrating surface in the first period of oscillation is calculated as

$$\Delta m_1(0) = \overline{m}(0)(b(0)d(0))^{-3} A, \quad (17)$$

and the average velocity of particle impact with the vibrating surface for the first period of oscillation can be calculated as

$$\Delta V = \frac{1}{t_0} \int_0^{t_0} 2\pi A \omega \cos(2\pi\omega t) dt, \quad (18)$$

where $t_0 = 0.25/\omega$ is the temperature distribution and dispersion pressure $p_1(y) = \text{const}$ obtained for the first period of oscillation are used to determine the dilatancy $\bar{\varepsilon}_1(y)$ and void fraction distributions $\varepsilon_1(y)$ based on equation of state (1).

In all subsequent periods of vibration oscillation, the quasi-thermal flux $Q_i(y)$ and he granular temperature profile $\theta_i(y)$ are determined using a similar scheme, but taking into account the inhomogeneous distribution of the void fraction and, accordingly, the particle concentration obtained during the previous period using the previously presented expressions (15) and (16). When calculating the quasi-thermal flux generated by a vibrating surface $Q_i(0)$, information on the average particle fluctuation velocity $\langle V_i'(0) \rangle$ in the volume of the medium adjacent to the surface is required for each subsequent period. This velocity is calculated using the local temperature of the granular medium $\theta_{i-1}(0)$ obtained in the previous period, i.e., it is assumed that, according to (3)

$$\langle V_i'(0)\rangle = (2\theta_{i-1}(0)/\rho_{i-1}(0))^{0.5}. \quad (19)$$

The simulation is carried out until certain minimum permissible changes in the temperature profile are reached. Implementation of the described algorithm determines the distribution of the solid phase fraction and the kinetic energy of fluctuations of uniform particles of the granular medium under the influence of vibrational oscillations in microgravity conditions.

2.4. Modeling the effects of quasi-diffusion separation and clustering of particles of a granular medium under the influence of vibrational oscillations in microgravity conditions

Under the heterogeneity of the solid phase distribution in the gaseous state of solid particles, mass transfer fluxes are formed, leading to the redistribution of the nonuniform components of the mixture [25, 26]. Separation fluxes are formed due to differences in the velocities of quasi-diffusion movements of nonuniform particles. During impact interactions of such particles under conditions of a solid phase concentration gradient, particles with a high fluctuation velocity (less dense, smaller, more elastic and smooth) experience an excess momentum directed along the void fraction gradient [25, 26]. As a result, in a granular medium that is inhomogeneous in composition and structure and in a gaseous state of solid particles, the effect of quasi-diffusion separation occurs. Quasi-diffusion separation occurs under the action of a driving force defined as the relative magnitude of the gradient of the average distance s between particles $\partial \ln s(y)/\partial y$. As a result, the intensity of the quasi-diffusion separation flux of particles with a high fluctuation velocity is determined by the following expression [25, 26]

$$j_s = c(y)\rho_b(y)D_{\text{ds}}\frac{\partial \ln s(y)}{\partial y}, \quad (20)$$

where $c(y)$ is the concentration of the control component; ρ_b is the local value of the bulk density of the medium; D_{ds} is the quasi-diffusion separation coefficient; $s = (b/b_0 - 1)d(y)$ is the average distance between particles; b_0 is the geometric parameter b calculated (13) for the case $\varepsilon = 0.2595$ (dense hexagonal packing of particles).

The intensity of quasi-diffusion transport of particles is determined by the velocity of their fluctuations and the mean free path [35]. As established in [36], the mean free path of particles in a granular medium, in general, depends not only on the properties of the particles and their volume fraction, but also on the hydrodynamic conditions in the volume of the medium. In a one-dimensional problem, it is possible to exclude the influence of boundary conditions on the dynamics of particle displacements and, as a consequence, to neglect the influence of shear effects on the mechanism of their collisions. Under this condition, the coefficient of quasi-diffusion separation of particles by size and density is expressed as follows [36]

$$D_{\text{ds}} = \frac{(\overline{m}(c)\langle V'(y)\rangle)^2}{2\overline{F}}\left(\left(\frac{\overline{d}}{m_1 d_1}\right)^2 - \left(\frac{\overline{d}}{m_2 d_2}\right)^2\right), \quad (21)$$

where $\overline{m}(c)$ is the average value of the particle mass; m_i, d_i is the mass and diameter of the particles of the i-th component; $\overline{d}$ is the average local value of the particle diameter; $\overline{F} = \langle V'(y)\rangle / s(y)$ is the average value of the particle collision frequency.

To model the distribution dynamics of nonuniform particles $c(y, t)$ within a granular medium bed subjected to vibrational oscillations under microgravity conditions, a general equation for separation dynamics [25, 26] was used. In the absence of convection and gravitational segregation fluxes, this equation is written as follows:

$$\frac{\partial(c\rho_b)}{\partial t} = \frac{\partial}{\partial y}\left[\rho_b\left(D_{\text{dif}}\frac{\partial c}{\partial y} - cD_{\text{ds}}\frac{\partial \ln s}{\partial y}\right)\right]. \quad (22)$$

In the presented form, the dynamics of the distribution of particles of the control component of the mixture $c(y, \tau)$ is described as the result of the conjugation of quasi-diffusion flows of mixing and separation of nonuniform particles. The mixing flow intensity is defined [25, 26] as the product of the quasi-diffusion mixing coefficient and the concentration gradient of the control component. The quasi-diffusion mixing coefficient is calculated based on the local dilatancy $\overline{\varepsilon}(y)$ and temperature $\theta(y)$ of the vibro-fluidized granular medium, which are mutually correlated, as determined by the equation of state of the granular medium (1). Dilatancy and temperature of a granular medium are used, in accordance with their definitions (2) and (3), to express local values of the average distance between particles $s(y)$ and the average velocity of their fluctuations $\langle V'(y)\rangle$, followed by the calculation of the quasi-diffusive mixing coefficient

$$D_{\text{dif}} = 1/3\langle V'(y)\rangle s(y). \quad (23)$$

The distribution dynamics of nonuniform particles in a bed of a binary particle mixture subjected to vibrational oscillations under microgravity conditions was modeled by numerically integrating equation (22) using the Crank-Nicolson difference scheme [37]. The solution was obtained under the boundary condition

$$D_{\text{dif}} \frac{\partial c}{\partial y} = cD_{\text{ds}} \frac{\partial \ln s}{\partial y} \bigg|_{y=0,\delta/2} = 0 \qquad (24)$$

and an initial condition corresponding to a uniform distribution of control particles within the bed volume

$$c(y,0) = \bar{c}, \qquad (25)$$

where $\bar{c}$ is the average concentration of control particles in the mixture. A uniform initial distribution of cohesionless particles is assumed due to the assumption of an entropic process of equalization of the parameters of a rarefied granular medium under zero-gravity conditions.

3. Results and Discussion

The study was carried out using mathematical modeling of the structural and kinematic parameters and distribution of bronze balls with diameters of 1 and 2 mm in an unbounded (one-dimensional) flat bed 45 mm thick under the influence of vibration in the absence of gravity. The bed characteristics and the vibration parameters of the surfaces bounding it were assumed to be equal to the corresponding characteristics of the measuring cell in the second experiment [2] (see the “Materials and Methods” section). The modeling was performed under the assumption that the vibrating surfaces were made of bronze and their physical and mechanical properties corresponded to those of the particles (restitution coefficient $k = 0.9$, friction coefficient $\mu = 0.4$ [2]). The concentration of control large particles in the mixture ($c = 0.561$ kg·kg^{-1}) and the average value of the solid phase fraction in the bed volume (0.089) were determined based on the quantitative ratio of large (80 pieces) and small (500 pieces) particles and their total volume in the volume of the measuring cell under the conditions of Experiment 2 [2].

The mathematical modeling of the effects of clustering and particle separation by size was carried out using a method for determining structural and kinematic characteristics based on the equation of state of a granular medium (1) and a mathematical model of the dynamics of mass transfer processes for nonuniform particles (22). Equation (1) is used in combination with the models of generation (7) and dissipation (15) of quasi-thermal flux.

The coefficient χ of the equation of state of a granular medium (1) was determined by modeling the distribution of 80 uniform large ($d = 2$ mm) bronze particles in a measuring cell in suborbital experiment No. 1 [2]. It was found that the distribution of the solid phase and the corresponding dilatancy values $\bar{\varepsilon}(y)$ in the measuring cell is modeled with a coefficient χ in equation (1), which is close to 1. The resulting value $\chi = 1$ was used to model the dynamics of the solid phase distribution and the granular temperature under the influence of vibrational oscillations in a mixture of nonuniform particles.

Figure 2 shows the distributions of $\varepsilon(y)$ and $\theta(y)$ in the particle mixture bed. Particle vibrations are generated by harmonic surface oscillations with a coordinate of $y = 0$. The presented results were used to determine the vibration parameters that facilitate particle cluster formation in the absence of gravity (Fig. 3). Furthermore, the results were used to model the concentration distribution of large particles along the bed thickness (Fig. 4).

In order to identify the conditions facilitating the formation of a particle cluster, Figure 3 shows the dependences of the gradient of the solid phase fraction and the granular temperature of the medium on the penetration depth of vibration oscillations into the particle bed. A comprehensive analysis of these dependences, combined with visual experimental information (the Parabolic Flight Campaign (PFC64) study) [2], which clearly demonstrates the cluster boundaries, suggests that the formation of a particle cluster is evidenced by a sharp increase in the gradient of the solid phase fraction at a bed depth of 14 to 18 mm, followed by a sharp decrease in its values.

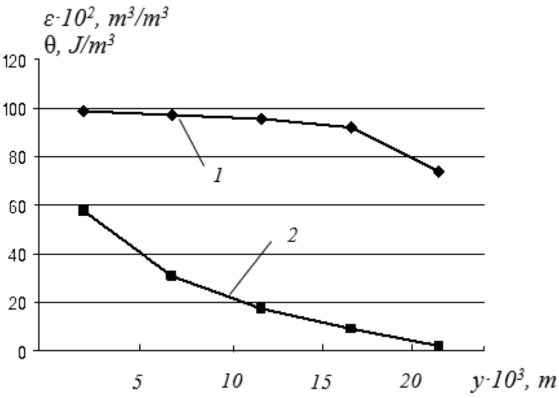

Fig. 2. Distribution of the volume fraction of voids *1* and the temperature of the granular temperature *2* in a bed of vibro-fluidized binary mixture of spherical bronze particles with diameters of 1 and 2·10^{-3} m in the absence of gravity

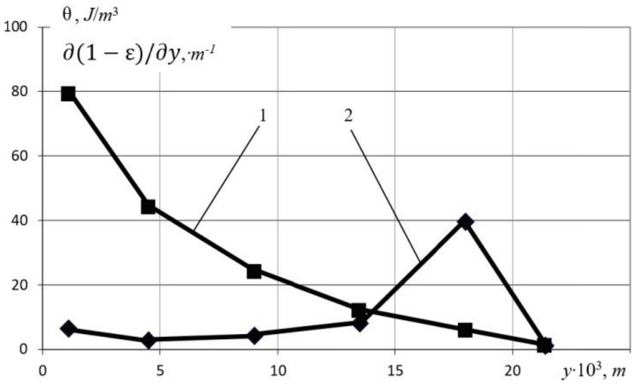

Fig. 3. Illustration of the physical conditions for particle cluster formation under vibrational action on a bed of a binary mixture of spherical bronze particles with a diameter of 1 and $2 \cdot 10^{-3}$ m in the absence of gravity: the temperature of the granular medium *1* and the gradient of the solid phase fraction *2* as functions of the coordinate in the direction of vibrational oscillations

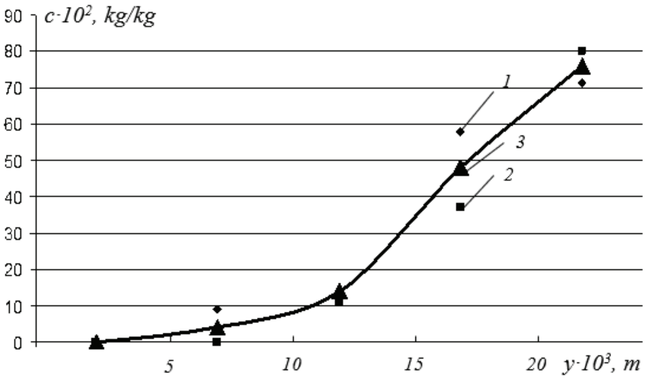

Fig. 4. Concentration distribution of large particles ($2 \cdot 10^{-3}$ m) mixed with small particles ($1 \cdot 10^{-3}$ m) in a vibro-fluidized bed of a binary mixture of spherical bronze particles under microgravity conditions: *1* – modeling results; *2* – experimental visual information [2]; *3* – trend line

Moreover, it is quite logical that the formation of a particle cluster occurs as the granular temperature approaches zero. As a result, the formation of a particle cluster is confirmed by a sharp increase in the gradient of the solid phase fraction at its boundary and low, close to zero, values of the gradient and granular temperature in the cluster volume. In this regard, it can be concluded that the main conditions for cluster formation during vibration exposure to a particle bed in a state of weightlessness are sufficiently high values of solid phase concentration and bed thickness, at which the quasi-thermal vibration flux has a limited penetration area. Consequently, a region forms within the particle bed in which the quasi-thermal vibration flux is close to zero.

The result of modeling the particle separation process by size during vibration exposure to a bed of their binary mixture under microgravity conditions is shown in Figure 4 in comparison with the experimental data obtained in [2]. The experimental data represent the result of averaging visual information regarding the distribution of large and small particles in the volume of the measuring cell presented in [2]. The averaging of visual information was carried out using a counting statistical method from fragments of the image of the measuring cell in Experiment 2, located to the left and right of its symmetry axis. Despite the noticeable discrepancy between some local calculated and experimental values, the concentration fields obtained by the modeling method based on equation (22) and experimentally [2] correspond to the trend line 3 common to them. One explanation for the presence of areas with a noticeable discrepancy between the calculated and experimental results in the concentration distributions may be the influence of the edge effect of the side walls of the measuring cell on the particle distribution under the experimental conditions. If the particle separation modeling was carried out under conditions of an unbounded bed (one-dimensional problem), then the relatively small distance between the side walls of the measuring cell compared to the particle size in the experiment [2] indicates the presence of features of a three-dimensional object.

A comprehensive analysis of the calculated and experimental concentration distributions shown in Figure 4 demonstrates that a phenomenological description of the vibroseparation process under microgravity conditions can be achieved using the quasi-diffusion separation effect [25, 26, 36]. Furthermore, based on the findings, it is possible to predict the high potential for technological application of the quasi-diffusion separation effect under microgravity conditions. This primarily applies to technologies for separating materials based on a range of particle physical and mechanical properties (size, density, roughness, and elasticity). The prospects for the technological use of the effect of quasi-diffusion separation under microgravity conditions are determined, on the one hand, by the wide possibilities of organizing intensive and stable quasi-diffusion interaction of nonuniform particles in a rarefied state (low proportion of solid phase) and, on the other hand, by the significant dependence of the rate of quasi-diffusion of particles on their properties.

4. Conclusion

Based on a phenomenological analysis of the state of a granular medium subjected to vibration in microgravity, an explanation for the physical nature of the phenomena and a method for predicting the effects of cluster formation and distribution of nonuniform particles have been proposed. The equation of state for a granular medium has been adapted to describe the structural and kinematic characteristics of a granular medium consisting of cohesionless spherical particles subjected to vibration in the absence of gravity. The conditions for cluster formation in a rarefied particle bed under the influence of vibrations of the surfaces bounding the bed have been determined. It has been established that cluster formation is due to a limited region of active penetration of the quasi-thermal vibration flux into the bed and the presence of a region with a quasi-thermal flux close to zero. A phenomenological description of the vibroseparation process in microgravity conditions is provided by analyzing the interaction of particles in a vibrofluidized bed based on the fundamental principles of diffusion kinetics. It has been established that the dynamics of the concentration distribution of nonuniform particles can be described analytically as the conjugation of fluxes initiated by quasi-diffusion effects of separation and mixing. A comparison of the modeling results with experimental data obtained during a suborbital flight (Parabolic Flight Campaign (PFC64)) using the VIP-Gran instrument [2] indicates good qualitative agreement.

5. Conflict of interest

The authors declare no conflict of interest.

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