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Mathematical Modeling of Nonstationary Heat Processes of Thermal Treatment of Metal Products with Enhanced Hardness

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Abstract

The processes of thermal treatment of metal products are carried out on electrothermal setups, the main elements of which are a hydraulic press and heating plates between which a processed product is placed. The main problem of temperature treatment of products during these processes is to provide uniform heating of a product, i.e. constant temperature throughout its volume. Most often it is solved by experimental selection of power and arrangement of heaters in press plates. In this paper we propose a solution on the basis of mathematical modeling based on obtaining temperature fields of "a heating plate – a product" systems for any time. Mathematical models of non-stationary thermal processing a single heating plate and stabilizing its operating temperature have been developed. The universal numerical method for calculating the allowable surface power of the ohmic heater that is applicable to any system (transparent and opaque to thermal radiation) of an arbitrary shape has been proposed. The method for solving the mathematical model of automatic stabilization of heating plate temperature in ANSYS software allowing for the changes in the value of the time step has been developed. The procedure for calculating the heating plates insulation which takes into account the effect of the press elements on plate temperature is presented. The problem of thermal calculation of press plates has been formulated and the technique of its solution based on the experiment design theory has been proposed.

Keywords

Heating plate; mathematical modeling; Ohmic heater; thermal treatment, press equipment.

Introduction

The final stage of manufacturing products made of special steel alloys characterized by high values of strength and hardness is usually thermal treatment aimed at removing internal stress and “upgrading” product properties to the required values.

A promising method for thermal treatment of such materials is heating and loading holding while changing temperature in time according to regulations. For this purpose, a product is placed in a hydraulic press designed for thermal treatment of products made of metals and alloys. The main element of the heating press system is a heating plate in which the heating elements are available: channels for steam or slots to place inductors or resistors.

Most of the studies on the problems of thermal treatment of metal products are experimental in nature and devoted to the influence of various operating characteristics of the process on the microstructure and properties of the products of different geometry [1].

At the same time the quality of processed products are greatly affected by temperature conditions of metalworking processes under pressure. For example, Maeno et al. [2] studied the formability of high hardness materials for manufacturing products by hot stamping. The authors optimize the process by preventing the product temperature from a decrease through the use of supplementary devices.

The problems of developing and optimizing heating systems of the equipment for thermal treatment of metal products are virtually not considered in scientific literature.

Some of the results of modeling temperature fields in the elements of the press heating system and the choice of optimal design parameters of heating plates of steam and induction heating are dealt with in [3–7]. The present paper focuses on the same problems for press plates with ohmic (resistive) heaters.

Mathematical Model of Ohmic Heating of Press Equipment

Traditionally, the quality of products manufactured with the use of press equipment is thought to be almost completely determined by the degree of non-uniformity of a temperature field in the volume of products and depends on the choice of the type of heaters, heating plate design and molds. As a rule, R&D departments of industrial enterprises have to provide the required degree of non-uniformity of temperature fields of working surfaces of heating plates (from ± 5 to ± 1 °C). It is assumed that in this way the uniform heating of molds and products will be ensured and, as a result, the manufacture of a wide range of products will be possible.

While developing mathematical models of ohmic heating of press equipment, we made an assumption about uniform distribution of heater power in the volume of plate slots, i.e., specific heat emission of the heater (in W/m^3) is represented in the form:

$$q(x, y, z) = \begin{cases} \dfrac{I_i^2 R_i}{V_i}, \ \text{if}\ (x, y, z) \in \Omega_i, \ i = 1, ..., n; \\ 0, \ \text{otherwise}, \end{cases} \quad (1)$$

where R_i is resistance of the i-th heater, Ohm; Ω_i is a set of points coordinates belonging to the slot of i-th heater; V_i is a slot volume for the i-th heater, м^3; $I_i = P_i / U_i$ is current strength flowing through the i-th heater, A; P_i is power of the i-th heater, W; U_i is voltage, applied to the i-th heater, V.

The required diameter of the heater wire is defined by the power pre-selected from the condition of providing the desired heating rate [8]:

$$d = \sqrt[3]{\frac{4P^2}{\gamma_h U^2 W \pi^2}}, \quad (2)$$

where γ_h is specific electrical conductivity of the heater material, Ohm^{-1}m^{-1}; W is surface power of the heater,W/m^2.

Assuming the practical absence of heat transfer from the heater to the plate by convection and heat conduction, the heater surface power can be determined according to the formula

$$W = W_{id} \alpha_{rad}, \quad (3)$$

where α_{rad} is dimensionless coefficient of efficiency of the heater radiation; W_{id} is the surface power of the ideal heater, forming parallel infinite planes with the heated body.

According to the Stefan-Boltzmann law:

$$W_{id} = 5.67 \cdot 10^{-8} \varepsilon_{p-id} (T_h^4 - T_s^4), \quad (4)$$

where T_h is the heater temperature, K; T_s is the temperature of the heated body, K, taken as equal to the estimated maximum temperature of the plate slot wall; $\varepsilon_{p-id} = \left(\varepsilon_h^{-1} + \varepsilon_s^{-1} - 1\right)^{-1}$ is the reduced power of emissivity of parallel infinite planes; ε_h, ε_s is the emissivity factor of the heater and the plate slot walls, respectively.

The process of propagation of heat in the heating plate is simulated by unsteady heat conductivity equation:

$$\frac{\partial T}{\partial \tau} = a \nabla^2 T + \frac{q}{c\rho}, \quad (5)$$

where $a = \lambda / c\rho$ is thermal diffusivity factor of plate material, m^2/s; $T = T(x, y, z, \tau)$ is temperature at the point of a plate volume with coordinates (x, y, z) at time point τ, °C; c, ρ, λ is specific heat capacity, J/(kg·K), density, kg/m^3 and heat conductivity of plate material, W/(m·K), respectively.

The initial condition for equation (5):

$$T(x, y, z, 0) = T_0, \quad (6)$$

where T_0 is ambient temperature, °C.

Heat emission from the external surfaces of the non-insulated heating plate is characterized by boundary conditions of the third kind:

$$-\lambda \left. \frac{\partial T}{\partial n} \right|_{\Omega_{pl\,r}} = \alpha_r \left(T_r - T_0\right), \quad r = 1, ..., 6, \quad (7)$$

where $\Omega_{pl\,r}$ is the r-th surface of a heating plate (working surface, cover, the ends); $\alpha_r = \alpha_r^k + \alpha_r^u$ – heat emission coefficient of the r-th plate surface, W/(m^2·K); α_r^k is the convective heat emission coefficient determined by the well-known criterion equation [9]; α_r^u is the radiant heat emission coefficient determined by the Stefan-Boltzmann law; T_r is the average temperature of the r-th plate surface (°C).

A perfect thermal contact is supposed to be between the heater and the plate:

$$T_{\rm ng}\left(x,y,z\right)=T_{\rm pl}\left(x,y,z\right); \tag{8}$$

$$\lambda_{\rm ng}\frac{\partial T_{\rm ng}}{\partial n_{\rm ng-pl}}=\lambda_{\rm pl}\frac{\partial T_{\rm pl}}{\partial n_{\rm ng-pl}}, \tag{9}$$

where $x, y, z \in \Omega_{\rm ng-pl}$, index "ng" relates to the heater, index "pl" relates to the plate, index (ng−pl) relates to the plate boundary and the slot for the heater and surfaces $\Omega_{\rm ng-pl}$.

A mathematical model (1)−(9) must be supplemented by the restriction of the value of the temperature of the working surface of the heating plate at the final moment of heating time period (not lower than the set value of $T_{\rm min}$):

$$T_{\rm w} \ge T_{\rm min}, \tag{10}$$

where $T_{\rm w} = T(x, y, 0, \tau_{\rm h})$, $0 \le x \le l_{\rm pl}$, $0 \le y \le s_{\rm pl}$; $\tau_{\rm h}$ is the set time of plate heating to temperature $T_{\rm min}$, s; $l_{\rm pl}$, $s_{\rm pl}$ are the length and width of the working surface of the plate, respectively, m.

A choice of the method for solving the mathematical model (1)−(10) was made in favor of the finite elements method (**FEM**), implemented by a system of finite-element analysis ANSYS.

To check the model adequacy, we used physical experimental data obtained by JSC "Plant Tambovpolimermash" during testing a new design of the heating plate, which provides an operating temperature of 550 °C. Temperature measurements were conducted at three points of the working surface of the plate using contact thermometer TK-5.09, equipped with a high-temperature probe. The scheme of the control points arrangement is shown in Fig. 1.

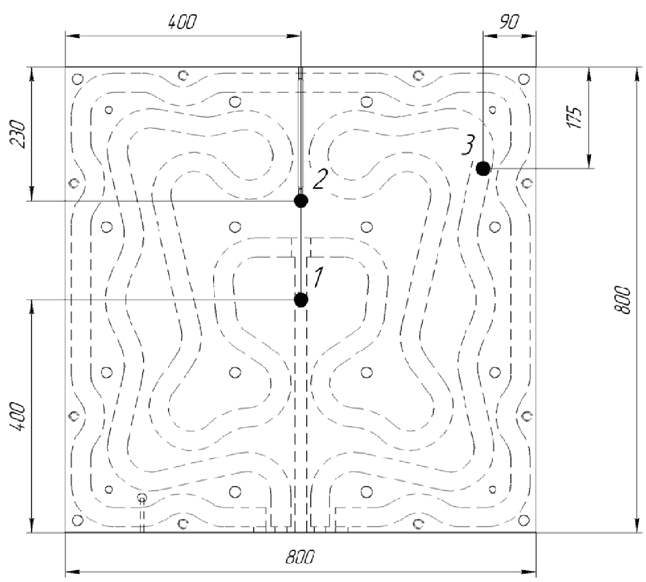


Fig. 1. Arrangement of thermocouples under experimental conditions

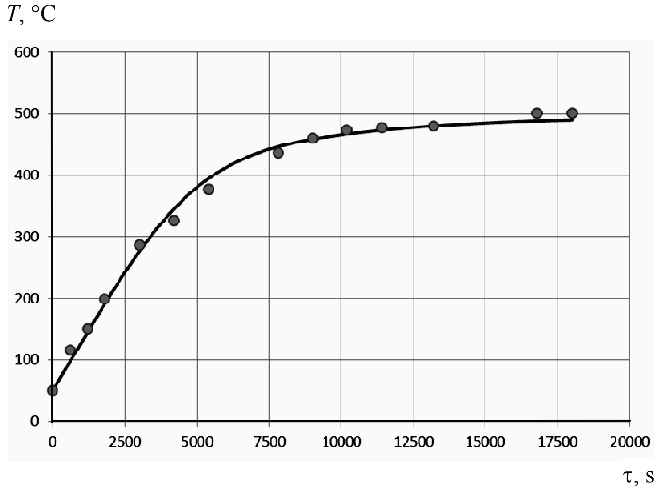


Fig. 2. Temperature at the location point of thermocouple No. 1:
——— – calculation; ● – experiment

The comparison of readings of thermocouple No. 1 and the results of the calculation of the plate temperature at the same point are shown in Fig. 2. The average deviation of the calculated values and the experimental data were as follows: for location point of thermocouple No.1 – 8.9 K or 1.5 %, thermocouple No. 2 – 13.6 K, or 2.1 %, thermocouple No. 3 – 16 K or 2.4 %.

To account for the presence of molds and products sandwiched between press plates, the model (1)−(10) must be supplemented by constraints on the placement of molds or products (if there are no molds) and boundary conditions (**BC**):

1) BC of the third kind for external surfaces of molds

$$\lambda_{\rm md}\frac{\partial T(x,y,z)}{\partial n_{\rm md-air}}-\alpha_7\left(T(x,y,z)-T_0\right)=0, \tag{11}$$

where α_7 is heat emission coefficient from the mold surfaces which are in contact with ambient air ($\Omega_{\rm md-air}$), W/(m^{2}K); $x, y, z \in \Omega_{\rm md-air}$; index "md" indicates the characteristics assignment to the mold.

2) BC of the fourth kind for the contact surface of the plate and the mold:

$$T_{\rm pl}\left(x,y,z\right)=T_{\rm md}\left(x,y,z\right), \tag{12}$$

where index "pl" indicates the characteristics assignment to the plate;

$$\lambda_{\rm pl}\frac{\partial T_{\rm pl}}{\partial n_{\rm pl\_md}}=\lambda_{\rm md}\frac{\partial T_{\rm md}}{\partial n_{\rm pl\_md}}, \tag{13}$$

where $x, y, z \in \Omega_{\rm pl\_md}$ (a contact surface of a plate and a mold);

3) BC of the fourth kind for the contact area of a mold and a product:

$$T_{\text{md}}\left(x, y, z\right) = T_{\text{rb}}\left(x, y, z\right), \quad (14)$$

where index "rb" indicates characteristics assignment to the product;

$$\lambda_{\text{md}} \frac{\partial T_{\text{md}}}{\partial n_{\text{md\_rb}}} = \lambda_{\text{rb}} \frac{\partial T_{\text{rb}}}{\partial n_{\text{md\_rb}}}, \quad (15)$$

where $x, y, z \in \Omega_{\text{md\_rb}}$ (contact surface a mold and a product).

If there are no molds (implementation of the processes of thermal treatment of metal products) in relations (12), (13) index "md" means the characteristics assignment to the product, and the conditions (14) – (15) are not used.

Surface Power Calculation of the Ohmic Heater

The surface power calculation of the ohmic heater according to formula (3) requires prior determination of the radiation efficiency coefficient value. The proposed approach to the calculation of this coefficient implies the determination of the total heat flow of heat absorbing surface F_2 (Fig. 3) according to formula:

$$Q = \int_{F_2} q dF_2 \quad (16)$$

To determine the surface power of the heater, the obtained value Q is divided by its surface F_1:

$$W = Q/F_1. \quad (17)$$

The numerical method used to determine the efficiency coefficient of ohmic heater radiation takes into account the presence of insulation and is based on the assumption of a small thickness of heat absorbing body (insulation), on the outer surface of which F_{ext} (Fig. 3) constant temperature is set. In this case, there will be a temperature gradient across the wall thickness, by means of which one can determine numerically (using FEM) the heat flow on surface F_2. The method implies fixed FEM calculation at given temperatures T_1 and T_{ext}, the calculation of the surface power of the heater according to (16) and determining the radiation efficiency coefficient (17).

To test the effectiveness of the method, there was made a comparison of its use results with analytical calculation results for axially symmetric system: a heating rod placed in the center of a square slot (Fig. 4). For such systems, the analytical calculation of the efficiency coefficient of the heater radiation is possible [10].

The accepted value of the emissivity factor of the heater is 0.8, the slot walls – 0.1, heater temperature – 1273 K, the slot walls – 773 K. Under these conditions, the analytical calculation gives value $\alpha_{\text{rad}} = 3.848$. The value of the radiation efficiency factor is greater than 1 because the area of heat absorbing surface in this case is much larger than the area of the heater: the created radiation heat exchange conditions are more favorable than those accepted for a perfect heater.

The total heat flow of heat absorbing surface, calculated in accordance with (16) in the ANSYS software amounted to 1468 W, the surface power of the heater according to (17) $W = 46730$ W/m^2. The surface power of the ideal heater according to formula (4) was $W_{\text{id}} = 12560$ W/m^2. According to formula (3) $\alpha_{\text{rad}} = 3.721$. Thus, the deviation of the calculation result using a developed numerical method from the result of analytical calculation was 3.3 %.

The proposed numerical method is characterized by versatility and can be used for systems that are opaque to thermal radiation, for example, when calculating a spiral isolated from the slot walls by a periclase. In this case, the concept of the radiation efficiency is meaningless, the problem lies in the direct determination of the allowable heater surface power by expression (17).

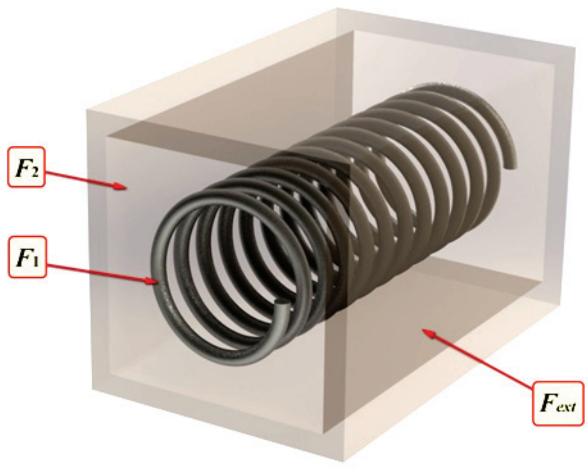


Fig. 3. Simulation of the system radiating bodies to calculate heat flows

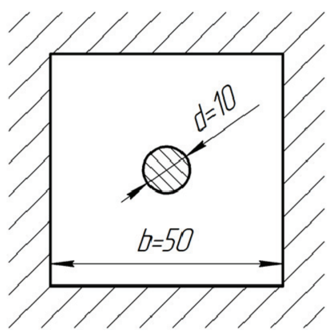


Fig. 4. Test model

Solution of the Mathematical Model

The basic operation mode of hydraulic presses used to implement thermal processes is the mode of automatic stabilization of the heating plates temperature. For non-stationary heat calculation of the plate in the mode of automatic temperature stabilization it is necessary to set the time step, the value of which will allow adequately simulating discrete changes in boundary conditions. The overestimated values of this step leads to the regulator response delay and, as a consequence, an inadequate description of the behavior of the research object. Using a small step increases the time of calculation.

To control the power of the heating plate elements with ohmic heaters, a PID-controller is often used, as a set point for the actuation of which T_{set} the set value of temperature T_{sensor} is applied. Moreover, value T_{set} can be an indication of one thermocouple or an average value of a few thermocouples. The output signal of the PID-controller is formed like this:

$$Y(\tau) = K_P e(\tau) + K_I \int_0^{\tau} e(\tau) d\tau + K_D \frac{de(\tau)}{d\tau}, \quad (18)$$

where K_P, K_I, K_D are amplification coefficients of proportional, integral and differential control signal components, respectively; $e(\tau) = T_{set} - T_{sensor}(\tau)$ is a temperature deviation in the place of setting the control thermocouple from value T_{set}.

Depending on the type of actuator, the output control signal is converted into the required form. For example, using pulse-width regulation Y represents a fraction of the maximum pulse duration of switching the solid state relay. The output signal in this case characterizes the fraction of the maximum power of heaters. Thus, for pulse-width regulation the following expression holds:

$$Y(\tau) = P(\tau)/P_m, \quad (19)$$

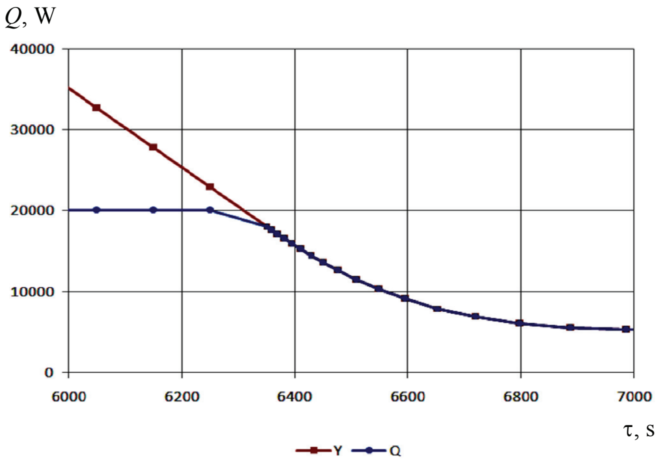
where $P(\tau)$ is the average over the pulse plate power, W; P_m is maximum plate power, W.

The minimum refresh period of the control signal of the PID-controller is limited by controllers response speed and can be tens of milliseconds, for example, for PID-controller TRM151 it is 0.3 s.

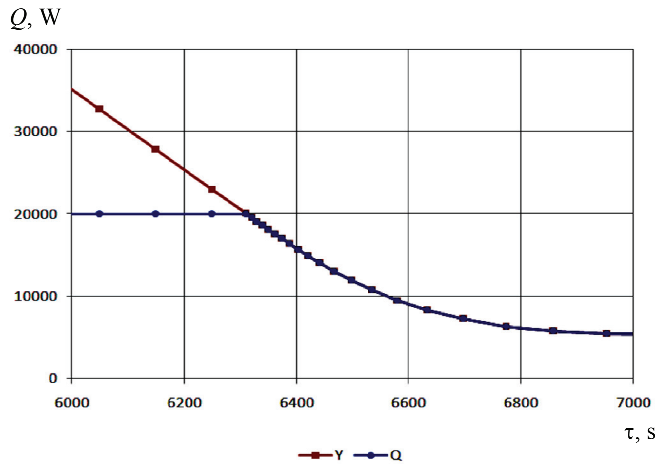
To accurately reproduce the system behavior in solving the mathematical model (1)–(15), it is necessary to use a small constant calculation time step due to the large amount of computation. It is proposed to change the value of the calculated time step in proportion to the absolute value of the derivative of control action: the value of the calculated step with sequence number n is defined by the following relationships:

$$\Delta_n = \min\{\Delta_{var\_pow}, \Delta_{smooth}, \Delta_{max\_pow}\}, \quad (20)$$

where $\Delta_{var\_pow} = \Delta_{max}\left(1 - \frac{|Q'|}{K_\Delta}\right) + \Delta_{min}$ is a step value calculated from the condition of the proportionality of the derivative of control action (Q); K_Δ is a scaling coefficient of control action; $\Delta_{smooth} = 2\Delta_{n-1}$ is a step value calculated from the condition of a smooth increase (the new value must not exceed twice the value of the previous one); $\Delta_{max\_pow} = \frac{Q_{max} - Y}{Y'}$, $Y > Q_{max}$, $Y' < 0$ is a step value calculated from the condition of the equality of the controller output signal to maximum control action. The latter condition is required to track by the extrapolation method the time point at which it is necessary to start reducing the level of control action. For clarity, Fig. 5 shows the calculated graphs of the controller output signal and control action without taking this condition into account (Fig. 5 *a*) and taking it into account (Fig. 5 *b*), respectively.



a)



b)

Fig. 5. The graphs of output signals of a controller and control action

As seen from the graphs, the maximum control action $Q_{\max}$ is limited by the value of 20 kW. In Fig. 5 *a* the calculated step changes with a delay which may cause an additional error. Fig. 5 *b* shows such a set step at which the controller output signal curve Y is predicted to intersect with a line corresponding to the maximum control action.

The absolute value of the derivative of the controller output signal at the calculated point n^* is accepted as K_Δ for which the following conditions are satisfied:

$$Y_{n^*} > Q_{\max}, \quad Y_{n^*+1} \le Q_{\max}. \tag{21}$$

In addition, a restriction in the form of inequality is imposed on Δn

$$\Delta_{\min} \le \Delta_n \le \Delta_{\max}. \tag{22}$$

Practical calculations showed that the use of the proposed method of changes in the value of the time step can reduce the duration of the calculation of temperature fields of the press heating system 6–8 times (from 30 to 4 hours.) compared with the use of maximum possible constant time step while maintaining the required accuracy of calculations.

The Problem of Thermal Calculation of the Press Plate

The problem of thermal calculation of the press plates with ohmic heaters is formulated as follows: to determine the number and power of the heaters, the material, the length and diameter of the wires of each of them, the slots geometry to accommodate them, which will provide the required plate heating rate up to the operating temperature, and the degree of temperature field non-uniformity of the working surface of the plate does not exceed the allowable one.

The difference between the maximum and minimum temperatures of the plate working surface is proposed to be used as optimality criterion:

$$\Delta = \max_{x,y} T\left(x,y,w,\tau_h\right) - \min_{x,y} T\left(x,y,w,\tau_h\right), \tag{23}$$

where $\delta \le x \le (l - \delta)$; $\delta \le y \le (s - \delta)$; l, s is the length and width of the plate, respectively, m; δ is the width of the plate edge which is not included in the working surface, m; $w = h$ is the plate height, m, if the working surface is its upper surface, and $w = 0$, if the working surface is its lower surface; τ_h is a given duration of the plate working surface heating from the temperature of ambient air T_0 to a given temperature T_z.

It is necessary to find the number of heaters n, the material, diameter d_i and length L_i of the wire of each of them, geometry $\mathbf{G}^{\langle i \rangle}$ of the slots for their placement, at which function (23) reaches a minimum value, and the following constraints are satisfied.

1. On the average temperature of the working surface of the plate:

$$\frac{1}{(l-2\delta)(s-2\delta)} \int_{\delta}^{l-\delta} \int_{\delta}^{s-\delta} T\left(x,y,w,\tau_h\right) dxdy \ge T_z. \tag{24}$$

2. On the geometric sizes of the slots for the heaters and their position in the plate:

$$\mathbf{G}_{\min}^{\langle i \rangle} \le \mathbf{G}^{\langle i \rangle} < \mathbf{G}_{\max}^{\langle i \rangle}, \; i = 1, \ldots, n, \tag{25}$$

$$V_i \cap V_j = \varnothing, \; i,j = 1,\ldots,n_h, \; i \ne j, \tag{26}$$

where $\mathbf{G}^{\langle i \rangle}$ is a slot parameters vector of the i-th heater (cross-sectional sizes, the coordinates of start, end and points of change in the direction, the radii of the fillets); V_i is the volume of the i-th heater slot with the inclusion of the neighborhood defined by constraints on the distance between the slots of adjacent heaters, the heater slot and the plate edges, m^3.

3. On the length of the wire of each heater:

$$\gamma_i \frac{U_i^2 \pi d_i^2}{4P_i} \le L_i \le \frac{Lp_i}{t_i} \sqrt{\left(\pi D_i\right)^2 + t_i^2}, \; i = 1, \ldots, n, \tag{27}$$

where D_i, t_i is the average diameter and coil pitch of the wire of the i-th heater, m; Lp_i is a slot length to place the i-th heater, m.

4. On the total power of the heaters which provides a desired rate of plate heating and compensates heat losses to ambient air:

$$\sum_{i=1}^{n_h} P_i \ge \frac{lsh\rho}{\tau_h} \int_{T_0}^{T_z} c(T) dT + \sum_{r=1}^{6} \alpha_r S_{\text{pl}\,r} \left(T_r - T_0\right), \tag{28}$$

where c, ρ is specific heat capacity, J·kg^{-1}·K^{-1}, and density of plate material, kg/m^3, respectively; $S_{\text{pl}\,r}$ is the area of the r-th surface of the heating plate: working surface, cover, edges, m^2; T_r is the average temperature of the r-th plate surface, °C, over a period of time $[0, \tau_h]$.

To determine the temperature field of the working surface of the plate, it is necessary to solve equation (5) under conditions (6)–(10). Specific heat emission of ohmic heaters is determined according to (1).

To solve the problem (23)–(28) using the relations (1), (5)–(10) and dependencies (16), (17) for determining the surface power of heaters, an algorithm was designed based on the use of the method of planning a computation experiment [11]. Using this method, we defined the configuration of the slots to place spiral heaters and their power for heating plate design providing an operating temperature of 550 °C. The calculations of heaters showed that nichrome X20H80, which is widely used for manufacturing electroheating coils, is not suitable in this case – the required coils lengths exceed the allowable ones for heating plates with a size of 800×800 mm (it is impossible to ensure condition (27)). Therefore, fechral X23Ю5T was chosen as a heater material, the working temperature of which is about 200 °C higher than that of nichrome X20H80.

Taking into Account the Presence of Thermal Insulation of the Heating Plate

Heating plates are in contact with a frame and a press table through heat insulation of the plates, which is to provide the temperature of the press table ≤ 90 °C that guarantees safe operation of the hydraulic system. To determine the temperature fields of thermal insulation plates and the press table, the proposed mathematical model of heating a single heating plate must be supplemented by the equations of heat distribution in heat insulation and press elements with boundary conditions of the third kind for external surfaces. The design of hydraulic presses is complex in terms of mathematical description of their geometry, therefore this supplement will manifold increase the amount of calculation. It is proposed to estimate the effect of press elements design on thermal processes in its heating system by means of special boundary conditions.

Let us consider the process of stationary heat conduction for a lower plate of thermal insulation, which, neglecting the non-uniformity of temperature field in the horizontal sections can be represented in the form of a flat wall (Fig. 6). Heat flow q through a flat wall with heat conduction λ is determined according to the expression:

$$q = \lambda \frac{(T_1 - T_2)}{h}. \tag{29}$$

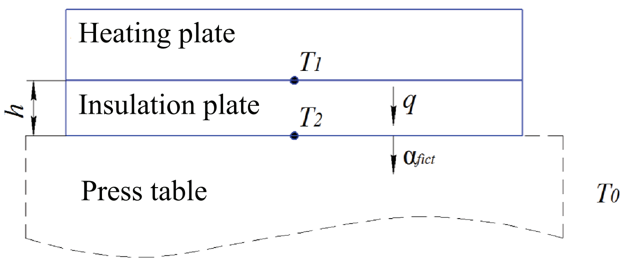


Fig. 6. Simulation of the press

Since the process is stationary, the entire power applied to the press elements dissipates to the environment. It is obvious that the power dissipated in the stationary state will increase with increasing temperature of the press in general and the temperature of table T_2 in particular. To simulate the effect of press elements on thermal processes in heating plates, it is proposed to use empty boundary conditions of the third kind, where the empty heat emission coefficient α_{fict} can be defined as the ratio of the heat flow q to the difference between the temperatures of press table T_2 and ambient air T_0:

$$\alpha_{\text{fict}} = \frac{\lambda (T_1 - T_2)}{h (T_2 - T_0)}. \tag{30}$$

To calculate empty values of heat emission coefficients, we conducted an experiment at JSC "Plant" Tambovpolimermash" aimed at determining stationary temperature T_2 for a plate with a power of 6 kW and a size of the working surface of 600×600 mm. The heating plate and the press table were separated by the plate made of asbestos-cement electro-technical arc-resistant material 40 mm thick. The results of the experiment are presented in Table 1.

Let us assume that the temperature derivative of the table is proportional to the difference between its stationary and current temperatures:

$$\frac{dT_2}{d\tau} = k (T_2 - T_{2c}), \tag{31}$$

where k is a proportionality factor (c^{-1}). The solution to differential equation (31) is the exponent:

$$T_{2c}(\tau) = T_2 - Ce^{kt}. \tag{32}$$

The function (32) can be regarded as an experimental-analytical model of a change in the press table temperature, therefore it is proposed to use this function as an approximating one. Thus, the problem consists in finding the values of constants T_2, C, k. To do this, we used Levenberg-Marquardt method [12], implemented by Mathcad software.

Table 1

The results of the experiment for determining the temperature of the press table

τ, min	60	90	120	150	180	210	240	270	300	330	360	390	420
T_2, °C	43	55	61	66	70	72	74	76	77	79	79	79	79

The obtained values of function coefficients (32), an empty heat emission coefficient calculated from formula (30) and the results of the extrapolation of the experimental data are presented in Fig. 7. As you can see, the graph of the approximating function satisfactorily reproduces the experimental data. Value α_{fict} = 18 W/(m^{2}K) rounded to the integer is adopted as an empty heat emission coefficient.

The results obtained can be also used for preliminary calculation of the multilayered insulating plate package. The total thermal resistance of the heat insulation layers is determined by the formula:

$$R_{\text{т}} = \frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2} + \ldots + \frac{h_{\text{ni}}}{\lambda_{\text{ni}}}, \tag{33}$$

where "ni" is the number of insulation layers.

For a stationary mode, the equality of a heat flow through a multilayer flat wall to a heat flow determined by empty heat emission coefficient holds true:

$$\frac{T_1 - T_2}{R_{\text{т}}} = \alpha_{\text{fict}}\left(T_2 - T_0\right). \tag{34}$$

Solving equation (34) with respect to T_2, we obtain:

$$T_2 = \frac{T_1 + \alpha_{\text{fict}} R_{\text{т}} T_0}{1 + \alpha_{\text{fict}} R_{\text{т}}}. \tag{35}$$

Thus, by using an empty heat emission coefficient one can determine the temperature of press design elements under stationary conditions, in particular, the temperature of its table, which should not exceed the allowable value of 90 °C. Equation (35) allows for reducing the amount of computation in the selection of materials and thickness of heat insulators from a few hours to a few seconds.

Results and Discussion

The mathematical model of heating a single plate of a hydraulic press with ohmic heaters (1)–(10), the adequacy of which is verified experimentally, allows you to promptly calculate (using ANSYS system and the proposed numerical method for determining the surface power of the ohmic heater) and estimate the degree of uniformity of the temperature field of the working plate surface.

Model (1)–(10) supplemented by relations (11)–(15) to account for the presence of molds and products sandwiched between press plates, as well as relation (19) modeling the output signal of a PID-controller, using pulse-width regulation allows to obtain temperature fields of plates, molds and processed products in the mode of automatic stabilization of heating plates temperature. The proposed method of changing the value of the time step in the process of model solution (1)–(15), (19) using ANSYS system allows to reduce the time of fields temperature calculation by order of magnitude compared with the maximum possible constant time step while preserving the required accuracy.

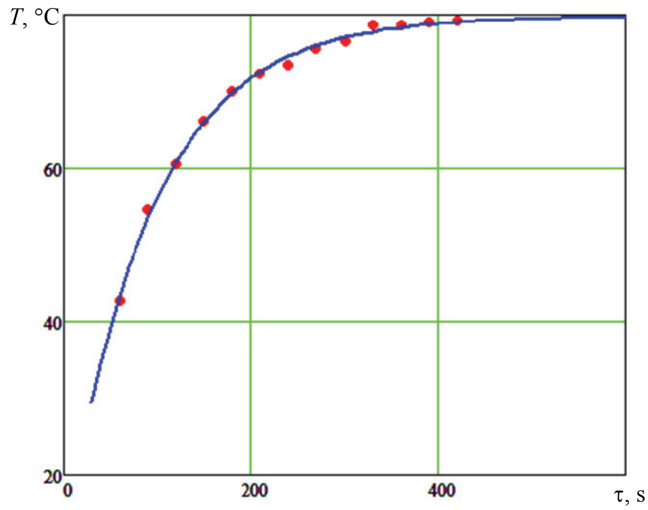


Fig. 7. Extrapolation of experimental data:
●●● – experimental calculation; ▬▬ – approximation results;
T_{2st} = 79.76 °C; C = 69.7; k = –0,011 c^{-1}; α_{fict} = 18.3 Вт·м$^{-2}$·K^{-1}

To determine the values of the parameters of the heating plate design (geometric sizes of slots for heaters and their position in the heater, the diameter and length of the wires of each of the heaters, the total power of the heaters which provides a required heating rate and compensates heat losses to ambient air), it is proposed to solve the optimization problem (23)–(28) using relations (1), (5)–(10) and dependencies (16), (17), the developed algorithm of solution that is based on the use of techniques for planning a computational experiment.

It is proposed to estimate the effect of heat insulation plates on thermal processes in the heating system of the press with special boundary conditions of the 3rd kind. To determine the values of empty heat emission coefficients, the experimental-analytical model of changes in the press table temperature was developed. The resulting relationship for determining the values of temperatures at the boundaries between thermal insulation plates, thermal insulation and the press table allows reducing the amount of calculation while selecting materials and thickness of heat insulators from a few hours to a few seconds without deterioration in the results quality.

Conclusion

Using the developed mathematical models, methods and algorithms, we have solved the problem of designing the heating system of a standard hydraulic vulcanizing press 250-8001E (single-plate, with a compression force of 2.5 MN and heating plates with a size of 800×800 mm, see [13]), designed for thermal treatment of rubber products and plastics. It is proposed to replace the press heating system including two plates of steel 45 with induction heaters (the working temperatures of 150–250 °C) with a new one which provides:

– the implementation of medium- and high-temperature tempering (the working temperatures of 350–550 °C) products of high hardness alloys under pressing with a force of 2.5 MN in order to relieve the stresses arising in their manufacture;

– carrying out the product processing cycle "heating-soaking-cooling" under given duration of operations with a required force;

– required temperature profile on the working surfaces of the heating plates and in the volume of products;

– table press temperature ≤ 90 °C, which guarantees safe operation of the hydraulic system.

The heating system of the proposed design was made at JSC "Plant Tambovpolimermash" where its successful testing was carried out. At present, the press is used at one of the enterprises of the military-industrial complex of the Russian Federation.

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